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Physics Letters A 362 (2007) 377–380
PHYSICS LETTERS A
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A proposal for the implementation of quantum gates with photonic-crystal waveguides

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Received 20 September 2006; accepted 18 October 2006

Available online 25 October 2006

Communicated by P.R. Holland

Abstract

Quantum computers require technologies that offer both sufficient control over coherent quantum phenomena and minimal spurious interactions with the environment. We argue that, photons confined to photonic crystals, and in particular to highly efficient waveguides formed by linear chains of defects, doped with atoms or quantum dots, can generate strong nonlinear interactions between photons allowing for the implementation of both single and two-qubit quantum gates. The simplicity of the gate switching mechanism, the experimental feasibility of fabricating two-dimensional photonic-crystal devices and the integrability of such devices with optoelectronic components offer new interesting possibilities for optical quantum-information processing networks.

PACS: 03.67.Lx; 42.50.Pq; 42.70.Qs

Keywords: Quantum gates; Cavity QED; Photonic crystals; Waveguiding

In order to perform a quantum computation one should be able to identify basic units of quantum information i.e., qubits, initialize them at the input, perform an adequate set of unitary operations and then read the output [1,2]. Here we show that these tasks can be efficiently performed using photons propagating along defect chains within photonic crystals. This type of linear defects (defect chains) are known as coupled-resonator optical waveguides (CROWs) [3,4] and provide almost lossless guiding, bending and coupling of light pulses at ultra small group velocities [5–7]. Qubits can be represented by the "dual rail" CROW, i.e. by placing a photon in a superposition of two preselected defect chains such that each chain represents the logical basis state, 0 or 1. Quantum logic gates are then implemented by varying the length and the distance between the CROWs and by tuning the refractive index in some of the defects using external electric fields and cavity QED type enhanced nonlinear interactions between the propagating photons [8–13]. We start with a sketch of the underlying technology followed by a more detailed description of quantum logic gates and conclude with the estimation of the relevant experimental parameters.

Photonic crystals (PCs) are inhomogeneous materials whose relative permittivity is a periodic function in space [14,15]. For wavelengths comparable to the period of the PC they can exhibit photonic band gaps, similar to the electronic band gaps of (atomic) semiconductors. One can also introduce point and linear defects within a PC. A point defect introduces a bound state of the electromagnetic field within the photonic band gap which can act as a high-Q cavity. Many point defects can be brought together to form the above mentioned CROWs. A light pulse which enters a CROW propagates through a tunnelling/hopping mechanism between neighboring defects allowing for a tight-binding-like description of the pulse propagation [3,4,16]. We

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0375-9601/$ – see front matter
doi:10.1016/j.physleta.2006.10.046

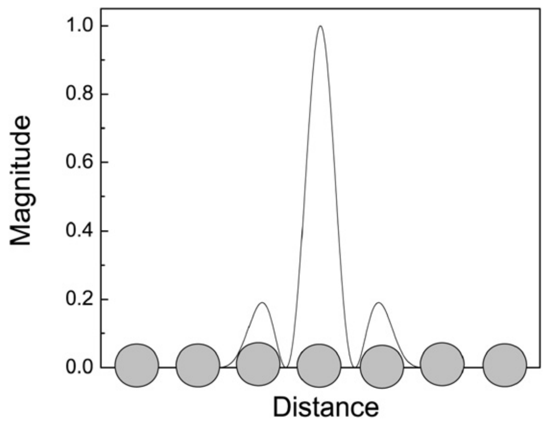

Fig. 1. Snapshot of a pulse propagating inside a CROW. The field intensity is mostly localized inside the defects of the CROW.

note that when a light pulse enters a CROW from free space, its spatial extent contracts to a least one order of magnitude while spending most of its time within a defect before tunnelling to its neighbor [7] (see Fig. 1).

After preselecting two CROWs and labeling them as 0 and 1 we can perform an arbitrary unitary operation on the resulting qubit by concatenating elementary single qubit gates such as the Hadamard gate and a phase shift gate. The Hadamard gate can be implemented by bringing two CROWs of the same qubit closer to each other, about one lattice constant apart, to allow photons to tunnel between them. This process, apart from phase factors, is equivalent to the action of a beam-splitter, or an optical coupler, in conventional optics. A single qubit phase gate can be implemented by increasing the length of one of the two CROWs; the resulting time-delay induces a relative phase shift.

As an example consider a single qubit interference, i.e. a sequence: the Hadamard gate, a phase gate, the Hadamard gate. It can be implemented by a device similar to the one shown in the lower part of Fig. 2, which is essentially a Mach–Zehnder interferometer (MZI) formed by defect chains in a PC. The two Hadamard gates correspond to the two areas in which the CROWs are brought closer to each other. Relative phase ϕ can be introduced by varying the length of one of the CROWs in the area between the two Hadamard gates. If a pulse of light is injected into one of the input ports it will emerge at the one of the two output ports with the probabilities $\sin^2(\phi/2)$ and $\cos^2(\phi/2)$, respectively, where ϕ is the accumulated phase difference between the two arms. This has been demonstrated experimentally for 2D CROW-based MZIs in the microwave regime [17] as well as for optical telecommunications wavelengths [18,19].

Although the existing experimental realizations of a CROW-based MZIs [17–19] have the phase shift ϕ fixed by the architecture, one can introduce an active phase control [20]. This can be achieved by placing a medium with tunable refractive index into one of the arms of the interferometer in between the Hadamard gates. Defects in one of the arms can be doped with atoms or quantum dots of resonance frequency ω_{ge}. These two-level systems can be then tuned to be on and off-resonance with the propagating light of frequency ω by applying an external electric field, i.e. by using the Stark effect. Initially the dopants are far off resonance with the light pulse allowing the pulse to enter the CROWs without any reflections. As soon as the pulse reaches the area in between the Hadamard gates the electric field is applied bringing the dopants closer to resonance and inducing a near-resonant dispersive interaction. When the detuning $\delta = \omega_{ge} - \omega$ is smaller than both ω and ω_{ge} and, at the same time, much larger than the coupling constant between the dopant and the light field Ω, i.e. when $\omega_{ge}, \omega \gg \delta \gg \Omega$, the combined dopant-light system acquires a phase proportional to $(\Omega^2/\delta)T$, where T is the interaction time. Both δ and T can be externally controlled and, this way, one can introduce any desired phase shift between the two arms of the interferometer.

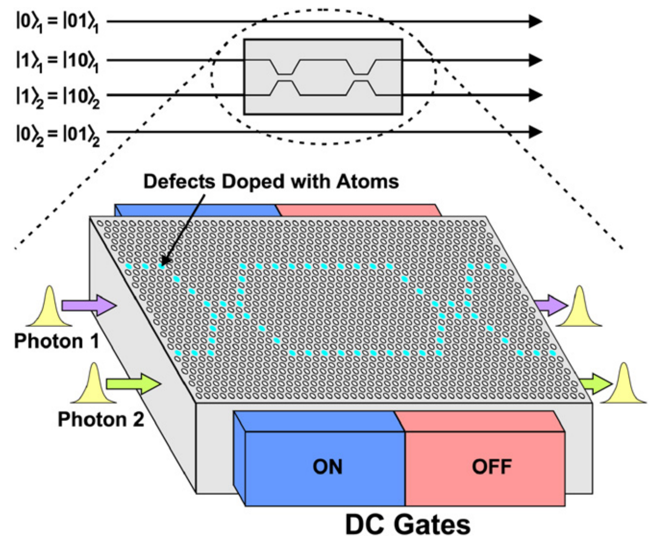

Fig. 2. The upper part shows a schematic of four CROWs which represent two qubits. The two central waveguides, belonging to two different qubits, are brought together in a nonlinear interferometric device which is shown below the schematic. The device is integrated onto a micrometer-sized 2D PC. The defect chains, shown in cyan, transfer photons from left to right. The two defect chains are brought closer to each other right after the entrance and before the exit of the device, allowing photons to tunnel between them. The defects in between these two regions are doped with atoms or quantum dots which can be tuned to be on- and off-resonance with the propagating light by applying an external electric field. An interplay between (resonant) two-photon and (dispersive) one-photon transitions leads to phase shifts required both for single-qubit phase gates and two-qubit controlled-phase gates. The blue and pink boxes mark the area where the electric field is on and off, respectively. When the field is switched on it induces a nonlinear phase shift. However, at the end of the quantum gate operation the field is selectively turned off to the right of the defect where the phase shift was induced, allowing photons to be released back to the propagating modes. (For interpretation of the references colour in this figure legend, the reader is referred to the web version of this article.)

Let us now show how the device shown in Fig. 2 can be used to implement a two-qubit conditional phase gate. The two qubits are represented by four CROWs labelled as $|0\rangle_1$, $|1\rangle_1$ and $|0\rangle_2$, $|1\rangle_2$ respectively for the first and the second qubit. Only two of the four CROWs enter the device. They have labels $|1\rangle_1$ and $|1\rangle_2$ and represent the binary 1 of the first and the second qubit. Thus the device operates either on vacuum (input $|0\rangle_1|0\rangle_2$), or on a single photon (inputs $|0\rangle_1|1\rangle_2$ and $|1\rangle_1|0\rangle_2$) or on two photons (input $|1\rangle_1|1\rangle_2$). The desired action of the device, i.e. the conditional phase shift gate, is: $|0\rangle_1|0\rangle_2 \to |0\rangle_1|0\rangle_2$, $|0\rangle_1|1\rangle_2 \to |0\rangle_1|1\rangle_2$, $|1\rangle_1|0\rangle_2 \to |1\rangle_1|0\rangle_2$, $|1\rangle_1|1\rangle_2 \to -|1\rangle_1|1\rangle_2$. This means that the device allows the

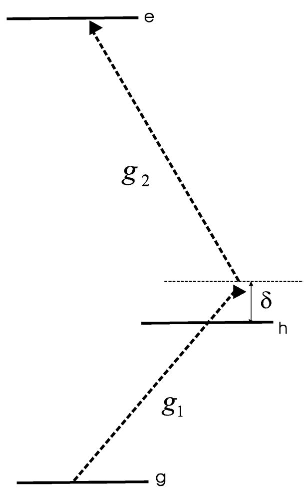

Fig. 3. The relevant energy levels of the dopants. A photon of frequency ω is equally detuned from ω_{gh} and ω_{he} ($\pm\delta$) and undergoes a dispersive interaction with the dopants. However, a two photon pulse is resonant with the energy separation between the levels g and e, i.e., $2\omega = \omega_{gh} + \omega_{he}$, and undergoes a resonant interaction.

vacuum and one-photon states pass through undisturbed and interact only with a two-photon state. We can achieve this by an interplay of dispersive interaction for single photons and resonant interactions for two photons.

Let us focus only on the CROWs modes that actually enter the device, i.e. $|1\rangle_1$ and $|1\rangle_2$, and consider their photon occupation numbers. From now on $|nm\rangle$ means n photons in mode $|1\rangle_1$ and m photons in mode $|1\rangle_2$. If no phase shift is induced, the device affects the transformation: $|00\rangle \to |00\rangle \to |00\rangle$, $|01\rangle \to (|01\rangle - |10\rangle)/\sqrt{2} \to |01\rangle$, $|10\rangle \to (|01\rangle + |10\rangle)/\sqrt{2} \to |10\rangle$, $|11\rangle \to (|20\rangle - |02\rangle)/\sqrt{2} \to |11\rangle$, where the first and the second arrow correspond to the action of the first and the second Hadamard gate, respectively. All we need is a nonlinear medium in between the Hadamard gates such that the states $|00\rangle$, $|01\rangle$ and $|10\rangle$ do not change, while the states $|20\rangle$ and $|02\rangle$ both acquire the same phase π.

Following our scheme for the tunable single qubit phase gate, let us now consider dopants with three-level configuration, i.e., with electronic levels g, h and e forming a cascade with transition frequencies ω_{gh} and ω_{he}. The two transitions couple linearly to the hopping photons through electro-dipole interactions, as shown in Fig. 3. We place the dopants in both arms of the interferometer. A photon of frequency ω is symmetrically detuned from ω_{gh} and ω_{he} so that $\delta = |\omega_{gh} - \omega| = |\omega_{he} - \omega| \gg g_1, g_2$, where g_1, g_2 are the corresponding coupling constants for the two transitions. Thus a single photon can only undergo a dispersive interaction with the dopants. However, a pulse with two photons is resonant with the energy separation between the levels g and e, i.e. $2\omega = \omega_{gh} + \omega_{he}$, and undergoes a resonant interaction. This can be quantified by the following effective Hamiltonian, extensively studied in the theory of micromasers [21],

$$H_{\text{eff}} = \frac{g_1^2}{\delta}\sigma_{gg}(a^\dagger a) + \frac{g_2^2}{\delta}(\sigma_{ee}aa^\dagger) + \frac{g_1 g_2}{\delta}(\sigma_{ge}a^{\dagger 2} + \sigma_{eg}a^2), \tag{1}$$

where $a^\dagger$, a are the photon creation and annihilation operators and $\sigma_{ij} = |i\rangle\langle j|$ with $i, j = g, h, e$ are the corresponding atomic operators. The first two terms describe the dispersive interaction and the third term the two-photon resonant interaction.

If the dopant is initially in level $|g\rangle$ then the joint dopant-field state evolves, after time t, to [21]

$$|g\rangle|00\rangle \to |g\rangle|00\rangle, \tag{2}$$

$$|g\rangle|01\rangle \to e^{-i\varphi}|g\rangle|01\rangle, \tag{3}$$

$$|g\rangle|10\rangle \to e^{-i\varphi}|g\rangle|10\rangle, \tag{4}$$

$$|g\rangle|20\rangle \to e^{2i\varphi}[\cos\kappa t|g\rangle|20\rangle + \sin\kappa t|e\rangle|00\rangle], \tag{5}$$

$$|g\rangle|02\rangle \to e^{2i\varphi}[\cos\kappa t|g\rangle|02\rangle + \sin\kappa t|e\rangle|00\rangle], \tag{6}$$

where $\kappa = g_1 g_2\sqrt{2}/\delta$, and $\varphi = (g_1)^2 t/\delta$. For $\kappa t = \pi$, the two-photon interaction completes a full Rabi oscillation, acquiring a total phase $\phi = \pi + 2\varphi$, where $\varphi = g_1\pi/(g_2\sqrt{2})$. The ratio $g_1/g_2 = 2\sqrt{2}$ yields $\varphi = 2\pi$ which means that the two-photon state acquires a minus sign while the remaining states are brought back to their originals. Under these conditions, the time-evolution showed above reproduces an instance of a two qubit conditional phase shift gate.

For an actual realisation of the photonic quantum-computation scheme described above, we need doped 2D PCs of high quality, strong dopant-photon coupling, and reliable single photon sources together with efficient photo-detectors. These requirements are within the limits of current technology. More specifically, single PC defects with very small leaking losses and quality factors of the order of 10^5–10^6 have already been realized in the laboratory [22]. Ultrafast nonlinear response phenomena and switching from active elements embedded in PC structures have already been reported for PC-based MZIs doped with InAs quantum dots [23,24]. The latter are actual realisations which are very close to the device we propose in this work (Fig. 2). Since we require a quality factor of 10^6 in our scheme, a typical time-scale for undisturbed coherent quantum operations must be of the order of $T_1 = 1$ ns. Both the phase shift operation and the two photon nonlinear phase shift can be performed within a time period which is shorter by at least one order of magnitude. The coupling constant g for the individual atom-photon coupling, for example for the D2 atomic transition (852 nm) of a doped atom of ^{133}Cs, is of the order 3×10^9 Hz [8]. The maximum induced phase is $\sqrt{N}g^2 T_1/\Delta$ where N is the number of dopants in the defects. If $\Delta \approx 3 \times 10^{10}$ Hz and $N \approx 100$ dopants, then the time required to induce any phase between 0 and π, is roughly 0.1 ns. Similarly for the two photon nonlinear phase shift; the two photon Rabi frequency is proportional to $\sqrt{N}g_1 g_2/\Delta$ and $g_1 \simeq g_2 \sim 3 \times 10^9$ Hz. With the same typical value of Δ we get the gate operation time to be of the order of 0.1 ns. We note that these figures can be improved either by adding more dopants to the defects making the coupling stronger or by fabricating defect cavities with higher quality factors. Note that for the case of quantum dots, dipole moments are larger than the atomic ones and they will therefore couple more strongly with the field. However tuning between dots in different defects might be a problem in that case. Lastly, the switching time of the external gates depends on the photon crossing time which is of the order of a nanosecond given that the group velocity in a CROW can be as low as 10^{-4} of the speed of light [6,7]. Therefore, the required switching of the external electric fields should be performed on a timescale from nanoseconds to tens of picoseconds which is within current technology [23,24]. We would also like to add here that the switching operation could also be induced by applying a slightly detuned laser field (AC Stark shift), coupled to some other atomic level which is far from both the hopping photon resonance and the atomic levels under consideration. The size of the accessible device area will then be reduced to the focus area of the pulse which could be of the order of the wavelength of light, i.e., a few microns.

Decoherence due to interaction of the atoms with the vibrations of the medium is expected to be negligible for the case of a suspended atom (or cold atomic cloud) inside or close to the surface of the defect. This could be achieved, for example, through the lowering of a trap on top of the defect. Another source of decoherence could stem from the presence of disorder in a CROW due to the imperfections caused by the lithographic technique used for the fabrication of the CROW. Also, losses are expected to occur when photons enter the device leading to a reduction of the number of successful phase shifts per input number of photons. On the other hand, the losses inside the device are minimal and most of the photons that transverse the device will be phased shifted. All loss mechanisms mentioned above result in a reduction of the number of input uncorrelated photons which transform to phase-shifted photons. This can be counterbalanced by increasing the rate of incident photons whenever possible or by integrating a single photon source within the waveguide. In the case of a complete network with many gates, some tuning of the individual emitters might be needed.

An implementation of our scheme requires good synchronization of photon pulses, single photon sources and very efficient single-photon detectors. These requirements are very similar to those needed for quantum computation with linear optical elements [25]. However, our scheme is much less demanding in terms of resource overheads per reliable quantum gate. Recent progress in the development of single photon sources indicate that the photonic quantum computation should be a realistic experimental proposition [26,27]. For example, one can use the same active elements used in Refs. [23,24], i.e., InAs quantum dots, to create single-photon sources within the PC [28]. A more detailed study of all possible error mechanisms for this scheme is under way and will appear elsewhere.

In conclusion, we have shown that photons propagating in CROWs can generate strong nonlinear interactions enabling the implementation of both single and two qubit quantum gates. The simplicity of the gate switching mechanism using global external fields, the feasibility of fabricating two-dimensional PC structures and CROWs with current technology and the integrability of this device with optoelectronics should offer new interesting possibilities for optical quantum information processing networks.

Acknowledgements

D.G.A. thanks E.N. Economou, S.N. Bose and A. Bychkov for helpful discussions and D.K.L. Oi for help with the graphs. M.F.S. thanks CNPq for financial support. This work was supported in part by the QIP IRC (GR/S82176/01), European IST-FET programme RESQ (IST-2001-37559), the EU FP6-FET Integrated Project SCALA, CT-015714, and the Cambridge-MIT Institute. V.Y. is supported by the Greek–Hungarian bilateral programme "Nonlinear and Quantum Optics in Photonic Band Gap Materials: Phenomena and Methods" of the Greek General Secretariat of Research and Technology.

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