Coherent phenomena in photonic crystals

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We study the spontaneous emission, absorption, and dispersion properties of a Λ-type atom where one transition interacts near resonantly with a double-band photonic crystal. Assuming an isotropic dispersion relation near the band edges, we show that two distinct coherent phenomena can occur. First the spontaneous emission spectrum of the adjacent free-space transition obtains "dark lines" (zeros in the spectrum). Second, the atom can become transparent to a probe laser field coupling to the adjacent free-space transition.

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I. INTRODUCTION

It has been well established over the years that spontaneous emission depends not only on the properties of the excited atomic system but also on nature of the surrounding environment, and more specifically on the density of electromagnetic vacuum modes. Purcell [1] was the first to predict that the rate of spontaneous emission for atomic transitions resonant with cavity frequencies could be enhanced due to the strong frequency dependence of the density of modes. Employing a similar idea, Kellner [2] showed that spontaneous emission in a waveguide may be suppressed below the free-space level due to the singular behavior of the density of modes near the fundamental threshold frequency of the waveguide.

The above studies attracted recent attention when it was realized that periodic dielectric structures could be engineered in such a way that gaps in the allowed photon frequencies may appear [3]. Since then, photonic-band-gap (PBG) materials exhibiting photon localization have been fabricated initially at microwave frequencies, and more recently at near infrared and optical frequencies. The study of quantum and nonlinear optical phenomena, in atoms (impurities) embedded in such structures, led to the prediction of many interesting effects [3]. As examples we mention the localization of light and the formation of single-photon [4–7] and two-photon [8] “photon-atom bound states,” suppression and even complete cancellation of spontaneous emission [9–12], population trapping in two-atom systems [12], the phase-dependent behavior of the population dynamics [13,14], the enhancement of spontaneous emission interference [15–17], modified reservoir induced transparency [18–21] and transient lasing without inversion [22], the occurrence of dark lines in spontaneous emission [23,24], and other phenomena [25–27]. In addition there is also current interest with regard to the feasibility of the observation of either the quantum Zeno effect [28] or the quantum anti-Zeno effect [29] in modified reservoirs, such as the PBG.

In this paper we study the spontaneous emission and the probe absorption and dispersion spectrum in a Λ-type system, similar to the one used in previous studies [10,13,18,22,23,25,30], with one of the atomic transitions decaying spontaneously in a double-band [17] PBG reservoir [Fig. 1(b)]. The existence of two bands in the density of states will allow for a more realistic description of the dynamics. In the case of previous studies, usually only one band was considered [Fig. 1(c)] [10,13,18,22,23,25,30], and the unphysical approximation of a flat, infinite density of states at the left band edge was made. We show that in our case the spontaneous emission spectrum can exhibit more “dark lines” (zeros in the spectrum at certain values of the emitted photon frequency) in the spontaneous emission spectrum in the free-space transition that in the single-band case. In addition, the atom becomes transparent to a probe laser field coupled to the free-space transition, and ultra-slow group velocities are obtained near the transparency window. The dependence of these phenomena on the width of the gap is derived, and the differences with the single-band case results is discussed [18,23].

This paper is organized as follows. In Sec. II we apply the time-dependent Schrödinger equation to describe the interaction of our system with the modified vacuum, calculate the spontaneous emission spectrum in the free-space reservoir, and discuss its properties along with the differences from the single-band case [10,11]. In Sec. III, using the time-dependent Schrödinger equation, we analytically calculate the steady-state linear susceptibility of the system. Results of the absorption and dispersion of a probe laser field in this system, and their comparison with the single-band case, are also presented in Sec. III. Finally, we summarize our findings in Sec. IV.

II. EQUATIONS AND RESULTS FOR THE SPONTANEOUS EMISSION SPECTRUM

We begin with a study of the Λ-type scheme, shown in Fig. 1. This system is similar to that used in previous studies [10,13,18,22,23,25,30]. The atom is assumed to be initially in state $|2\rangle$. The transition $|2\rangle\rightarrow|1\rangle$ is taken to be near resonant with a modified reservoir (this will later be referred to as the non-Markovian reservoir), while the transition $|2\rangle\rightarrow|0\rangle$ is assumed to be occurring in free space (this will later be referred to as the Markovian reservoir). The spectrum of this latter transition is of central interest in this section. The Hamiltonian which describes the dynamics of this system, in the interaction picture and the rotating wave approximation, is given by (we use units such that $\hbar = 1$),

\[\hat{H}_{\text{INT}} = \sum_{\text{emitted}} \left(\hat{a}^{\dagger}_\text{emitted} \hat{a}_{\text{emitted}}\right) + \sum_{\text{absorbed}} \left(\hat{a}_\text{absorbed}^{\dagger} \hat{a}_\text{absorbed}\right) + \sum_{\text{transmitted}} \left(\hat{a}_\text{transmitted}^{\dagger} \hat{a}_\text{transmitted}\right) + \sum_{\text{reservoir}} \left(\hat{a}_\text{reservoir}^{\dagger} \hat{a}_\text{reservoir}\right).\]
Here \( g_\kappa \) denotes the coupling of the atom to the modified vacuum modes (\( \kappa \)), and \( g_\lambda \) denotes the coupling of the atom to the free-space vacuum modes (\( \lambda \)). Both coupling strengths are taken to be real. The energy separations of the states are denoted by \( \omega_{ij} = \omega_i - \omega_j \), and \( \omega_\kappa (\omega_\lambda) \) is the energy of the \( \kappa (\lambda) \)th reservoir mode.

The description of the system is done via a probability amplitude approach. We proceed by expanding the wave function of the system, at a specific time \( t \), in terms of the “bare” state vectors, such that

\[
|\psi(t)\rangle = b_2(t)|2,\{0\}\rangle + \sum_\kappa b_\kappa(t)|0,\{\kappa\}\rangle + \sum_\kappa b_\kappa(t)|1,\{\kappa\}\rangle.
\]

Substituting Eqs. (1) and (2) into the time-dependent Schrödinger equation, we obtain

\[
i\dot{b}_\lambda(t) = \sum_\kappa g_\lambda b_\kappa(t)e^{-i(\omega_\lambda - \omega_\kappa)t} + \sum_\kappa g_\kappa b_\lambda(t)e^{-i(\omega_\kappa - \omega_\lambda)t},
\]

\[
i\dot{b}_\kappa(t) = g_\lambda b_\lambda(t)e^{i(\omega_\kappa - \omega_\lambda)t},
\]

\[
i\dot{b}_\lambda(t) = g_\kappa b_\kappa(t)e^{i(\omega_\kappa - \omega_\lambda)t}.
\]

We proceed by performing a formal time integration of Eqs. (4) and (5), and substitute the result into Eq. (3) to obtain the integrodifferential equation

\[
\dot{b}_\lambda(t) = -\int_0^t dt' b_\lambda(t') \sum_\kappa g_\kappa^2 e^{-i(\omega_\kappa - \omega_\lambda)(t-t')} - \int_0^t dt' b_\lambda(t') \sum_\kappa g_\kappa^2 e^{-i(\omega_\kappa - \omega_\lambda)(t-t')}.
\]

Because the reservoir with modes \( \lambda \) is assumed to be Markovian, we can apply the usual Weisskopf-Wigner result [3], and obtain

\[
\sum_\kappa g_\kappa^2 e^{-i(\omega_\kappa - \omega_\lambda)(t-t')} = \frac{\gamma}{2} \delta(t-t').
\]

Note that the principal value term associated with the Lamb shift which should accompany the decay rate term has been omitted in Eq. (7). This does not affect our results, as we can assume that the Lamb shift is incorporated into the definition of our state energies. For the second summation in Eq. (6), the one associated with the modified reservoir modes, the above result is not applicable as the density of modes of this reservoir is assumed to vary much more quickly than that of the free space. To tackle this problem, we define the kernel

\[
K(t-t') = \sum_\kappa g_\kappa^2 e^{-i(\omega_\kappa - \omega_\lambda)(t-t')}
\]

\[
\approx \beta^{3/2} \int d\omega \rho(\omega)e^{-i(\omega - \omega_\lambda)(t-t')},
\]

\( \rho(\omega) \) being the density of states per unit volume. Thus in Eq. (6), we replace the contribution from \( \kappa \) of the second summation by the above expression. The procedure described above also incorporates the coupling of the atom to the reservoir modes.

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**FIG. 1.** (a) displays a three level, \( \Lambda \)-type atomic system. The thick dashed line denotes the coupling to the modified reservoir (PBG), and the thin dashed line denotes the background decay. (b) shows the density of modes for the case of the single-band isotropic PBG model. (c) shows the density of modes for the case of the double-band isotropic PBG model.
with $\beta$ being the atom-modified reservoir resonant coupling constant. The above kernel is calculated using the appropriate density of modes $\rho(\omega)$ of the modified reservoir. Using Eqs. (7) and (8) in Eq. (6), we obtain

$$b_2(t) = -\frac{\gamma}{2} b_2(t) - \int_0^t dt' b_2(t') K(t-t'). \quad (9)$$

The long-time spontaneous emission spectrum in the Markovian reservoir is given by $S(\delta_\lambda) \propto |b_2(t \to \infty)|^2$, with $\delta_\lambda = \omega_\lambda - \omega_{20}$. We calculate $b_2(t \to \infty)$ with the use of the Laplace transform of the equations of motion. Using Eq. (4) and the final value theorem, we obtain the spontaneous emission spectrum as $S(\delta_\lambda) \propto |\lim_{s \to -i\delta_\lambda} B_2(s)|^2$, where $B_2(s) = \int_0^\infty e^{-st} dt' b_2(t')$ is the Laplace transform of the atomic amplitudes $b_2(t)$, and $s$ is the Laplace variable. This in turn, with the help of Eq. (9), reduces to

$$S(\delta_\lambda) \propto \frac{\gamma}{|-i\delta_\lambda + \gamma/2 + K(s \to -i\delta_\lambda)|^2}. \quad (10)$$

Here $\tilde{K}(s) = \int_0^\infty e^{-st} K(t)$ is the Laplace transform of $K(t)$, which yields, from Eq. (8),

$$\tilde{K}(s) = \beta^{3/2} \int_0^\infty d\omega \frac{\rho(\omega)}{s+i(\omega-\omega_2)}. \quad (11)$$

Therefore, in order to calculate the spontaneous emission spectrum in the Markovian reservoir, we need to calculate $\tilde{K}(s)$.

We consider the case of a double-band isotropic model of the photonic crystal, which has an upper band, a lower band, and a forbidden gap; the dispersion relation near the edges [17] is approximated by

$$\omega_\kappa = \omega_{g_1} - A_1(\kappa - \kappa_0)^2, \quad \kappa < \kappa_0,$$

$$\omega_\kappa = \omega_{g_2} + A_2(\kappa - \kappa_0)^2, \quad \kappa > \kappa_0,$$

with $A_n = \omega_{g_n}/\kappa_0^2$, $(n=1,2)$. Then the density of modes [Fig. 1(c)], reads

$$\rho(\omega) = \frac{1}{2\pi} \left[ \frac{1}{\sqrt{\omega_{g_1} - \omega}} \Theta(\omega_{g_1} - \omega) + \frac{1}{\sqrt{\omega - \omega_{g_2}}} \Theta(\omega - \omega_{g_2}) \right], \quad (13)$$

where $\omega_{g_n}$ is the gap frequency and $\Theta$ is the Heaviside step function. Then, from Eq. (11), we obtain

$$\tilde{K}(s) = \frac{1}{2} \left( \frac{\beta^{3/2} \sqrt{i}}{\sqrt{s+i\delta_{g_1}}} + \frac{\beta^{3/2}}{\sqrt{i\sqrt{s+i\delta_{g_2}}}} \right), \quad (14)$$

with $\delta_{g_n} = \omega_{g_n} - \omega_{21}$.

We use the formulas obtained above, and calculate the spontaneous emission for several parameters of the system. From Eqs. (10) and (14) it is obvious that the spectrum exhibits two zeros (i.e., predicts the existence of two dark lines) at $\delta_\lambda = \delta_{g_1}$ and $\delta_\lambda = \delta_{g_2}$. This is purely an effect of the density of modes of Eq. (14), and the non-Markovian character of the reservoir. In the case of a Markovian reservoir the spontaneous emission spectrum would acquire the well-known Lorentzian profile and no dark line would appear in the spectrum. The behavior of the spectrum is shown in Fig. 2. In Fig. 2(a) we show an asymmetric case where the right-hand side threshold detuning ($\delta_{g_2}$) is kept constant while the left-hand side threshold detuning ($\delta_{g_1}$) is changed. There are three peaks and two zeros in the spectrum, one pronounced peak in the center, and two lower peaks in the sides. As $\delta_{g_1}$ goes further to the sides the left-hand-side peak is suppressed and the spectrum approximates that of the single-band case [10,11] which is shown in Fig. 3. That is because the left-hand-side singularity in the density of the reservoir modes of Fig. 1(c) is far detuned from the atomic transition.

In Fig. 2(b) we show a symmetric spectrum, which is the case when the atom is placed in the middle of the gap. This spectrum also has two zeros and three peaks. In this case when the threshold detunings increase (i.e., the width of the
FIG. 3. The spontaneous emission spectrum $S(\delta)$ (in arbitrary units) for the single-band case and parameters $\gamma=1$ and $\delta=0$ (full curve), $\delta=1$ (dashed curve), and $\delta=-1$ (dot-dashed curve). All parameters are in units of $\beta$.

gap increases) the spectrum approximates a Lorentzian shape. This is the result of the complete cancellation of any emission in the modified reservoir, as there are no modes available for a large range of frequencies around the relevant atomic transition. We note that, in the case when both transitions occur in free space, the spectrum under consideration would still have a Lorentzian shape but the areal under the curve would have to be smaller as photons are emitted from the second transition as well [23].

III. EQUATIONS AND RESULTS FOR THE PROBE ABSORPTION-DISPERSION SPECTRUM

The aim in this section is to investigate the absorption and dispersion properties of our system for a weak probe laser field. As the probe laser field is assumed to be weak, the probability amplitude approach can be employed for the description of the system. The Hamiltonian of the system, in the interaction picture and the rotating-wave approximation, is given by

$$H=\Omega e^{i\delta t}|0\rangle\langle 2|+\sum_{\kappa}g_{\kappa}e^{-i(\omega_{\kappa}-\omega_{21})t}|2\rangle\langle 0|a_\kappa+H.c.$$  

(15)

Here $\Omega$ is the Rabi frequency (assumed to be real, for simplicity) and $\delta=\omega-\omega_{21}$ is the laser detuning from resonance with the $|0\rangle\rightarrow|2\rangle$ transition, with $\omega$ being the angular frequency of the probe laser field. We note that we are interested in the perturbative behavior of the system to the probe laser pulse; therefore, we can eliminate the Markovian decay modes using the method presented in Sec. II, and study the system using the following effective Hamiltonian:

$$H=\left[\Omega e^{i\delta t}|0\rangle\langle 2|+\sum_{\kappa}g_{\kappa}e^{-i(\omega_{\kappa}-\omega_{21})t}|2\rangle\langle 0|a_\kappa+H.c.\right]$$  

$$-\frac{\gamma}{2}|2\rangle\langle 2|.$$  

(16)

The wave function of the system, at a specific time $t$, can be expanded in terms of the “bare” eigenvectors such that

$$|\psi(t)\rangle=c_0(t)|0\rangle+e^{-i\delta t}|2\rangle+c_2(t)e^{-i\delta t}|2\rangle+\sum_{\kappa}c_\kappa(t)|1\rangle.$$  

(17)

and $c_0(t=0)=1$, $c_2(t=0)=0$, $c_\kappa(t=0)=0$. Substituting Eqs. (16) and (17) into the time-dependent Schrödinger equation and eliminating the vacuum amplitude $c_0$, we obtain the time evolutions of the probability amplitudes as

$$i\dot{c}_0(t)=\Omega c_2(t),$$  

(18)

$$i\dot{c}_2(t)=\Omega c_0(t)-c_2(t)-i\int_0^t dt' K'(t-t')c_2(t'),$$  

(19)

with the kernel

$$K'(t-t')=\sum_{K}g_K^2 e^{-i(\omega_{K}-\omega_{21}-\delta)(t-t')}K(t)e^{i\delta t}.$$  

(20)

All the coupling constants ($g_\kappa$, $\beta$, $\Omega$) are assumed to be real, for simplicity.

The equation of motion for the electric field amplitude $E(z,t)$ is given by [31]

$$\left(\frac{\partial}{\partial z}+\frac{1}{v_g}\frac{\partial}{\partial t}\right)E(z,t)=-i\frac{\omega}{2\epsilon}\chi(\delta)E(z,t),$$  

(21)

where $\chi(\delta)$ is the linear susceptibility of the medium, and $v_g=c[1+(1/2)\text{Re}(\chi)+((\omega^2/2)\partial\text{Re}(\chi)/\partial\omega)]$ is the group velocity of the laser pulse, with the derivative of the real part of the susceptibility being evaluated at the carrier frequency.

Since the transition $|0\rangle\rightarrow|2\rangle$ is treated as occurring in free space, the steady-state linear susceptibility is given by

$$\chi(\delta)=-\frac{4\pi N|\mu_0|^2}{\Omega(z,t)}c_0(t\rightarrow\infty)c_2^* (t\rightarrow\infty),$$  

(22)

with $N$ being the atomic density. Therefore, in order to determine the steady state absorption properties we have to solve Eqs. (18) and (19) for long times. We assume that the laser-atom interaction is very weak ($\Omega<<\beta,\gamma$) so that $c_0(t)\approx 1$ for all times, and, by using perturbation theory, Eqs. (18) and (19) reduce to

$$i\dot{c}_2(t)=\Omega -\left(\delta+i\frac{\gamma}{2}\right)c_2(t)-i\int_0^t dt' K'(t-t')c_2(t').$$  

(23)

We further assume that $\Omega(z,t)$ is approximately constant in the medium, and with the use of the Laplace transform we obtain, from Eq. (23),

$$C_2(s)=\frac{\Omega}{s\left[\delta+i\gamma/2+i\tilde{K}(s)+is\right]}.$$  

(24)
If \( \gamma \neq 0 \) then the terms inside the brackets of Eq. (24) have only complex, not purely imaginary, roots. Therefore we can easily obtain, using the final value theorem, the long-time behavior of the probability amplitude:

\[
\lim_{s \to 0} C_2(s) = \frac{\Omega}{\delta + i \gamma/2 + i \bar{K}(s \to 0)}
\]

Using Eqs. (22) and (25), we find that the steady-state linear susceptibility of our system is given by

\[
\chi(\delta) = -\frac{4 \pi N |\mu_{02}|^2}{\delta - i \gamma/2 - i \bar{K} \delta (-i \delta)}.
\]

Therefore, the absorption and dispersion of the probe laser field are determined by the density of modes of the non-Markovian reservoir. In our case the linear susceptibility becomes zero at \( \delta = \delta_1 \) and \( \delta = \delta_2 \); therefore, the medium becomes transparent to the laser field. This result is in contrast with the case in which the transition \( |2\rangle \leftrightarrow |1\rangle \) occurs in free space, where the well-known Lorentzian absorption profile is obtained. Typical spectra are shown in Fig. 4, where
only the left-hand-side threshold detuning is changed. Both the absorption and dispersion spectra are asymmetric, and their shapes depend critically on the detunings. We note that, as expected, Fig. 4(b) resembles, the corresponding spectra in the case of a single-band reservoir [18] which is shown in Fig. 5.

We also present the case of symmetric spectra in Fig. 6, where the detunings are symmetrically changed i.e., with the transition frequency of the atom in the middle of the gap. Here the interesting effect is that the transparency effect in the sides of the spectra is combined with the usual absorption or dispersion profile near the center of the spectrum.

We note that besides the transparency effect which occurs for the predicted values of frequency, the group velocity of the probe laser pulse is also effectively reduced near the transparency window due to the steepness of the dispersion curve. These results are related to that of electromagnetically induced transparency (EIT) [31–34] in a four-level laser-driven tripod scheme [35]. However, EIT occurs through the application of external laser fields. Here transparency is intrinsic to the system, as it occurs due to the presence of a square-root singularity of the density of modes at the band edges.

IV. SUMMARY

In this paper we studied the spontaneous emission, absorption, and dispersion spectra of a three-level atom embedded in a two-band isotropic photonic crystal. For spontaneous emission we have shown that the spectrum exhibits two dark lines and can be significantly modified via the system parameters. The explicit dependence of the spectrum on the width of the gap and on various values of the atomic parameters was also analyzed. For the absorption and dispersion spectra, two transparency windows exist. Near the transparency windows a very slow group velocity for the laser pulse is obtained. In this case too, the spectra depend critically on the atomic parameters and the width of the gap.

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