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PAPER

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Frozen photons in Jaynes–Cummings arrays

Nikolaos Schetakis[1,4], **Thomas Grujic**[2], **Stephen Clark**[2,3,4], **Dieter Jaksch**[2,3] **and Dimitris Angelakis**[1,3,4]

[1] Science Department, Technical University of Crete, Chania, Crete, 73100, Greece
[2] Clarendon Laboratory, University of Oxford, Parks Road, Oxford OX1 3PU, UK
[3] Centre for Quantum Technologies, National University of Singapore, 2 Science Drive 3, Singapore 117542

E-mail: nsxetakis@yahoo.gr, t.grujic1@physics.ox.ac.uk and dimitris.angelakis@gmail.com

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Abstract
We study the origin of 'frozen' photonic states in coupled Jaynes–Cummings–Hubbard arrays. For the case of half the array initially populated with photons while the other half is left empty, we show the emergence of a self-localized photon or 'frozen' states for specific values of the local atom–photon coupling. We analyse the dynamics in the quantum regime and discover important additional features that do not appear to be captured by a semi-classical treatment, which we analyse for different array sizes and filling fractions. We trace the origin of this interaction-induced photon 'freezing' to the suppression of the excitation of propagating modes in the system at large interaction strengths. We discuss in detail the possibility of experimentally probing the relevant transition by analysing the emitted photon correlations both in the idealized lossless case and more realistic scenarios when reasonable losses are included. We find a strong signature of the effect in the emitted photons statistics.

(Some figures may appear in colour only in the online journal)

1. Introduction

Since the first proposals to realize strongly correlated many-body states of light in resonator array architectures [1–3] a rapid growth of interest in these structures has been seen along several lines of investigation. Perhaps most recently, significant attention has been paid to finding novel observable non-equilibrium dynamical effects in modest-sized arrays, both in the steady state of driven-dissipative systems [4–9] and in explorations of coherent array dynamics [10–14]. Resonator arrays are in many ways ideal platforms for the exploration of non-equilibrium quantum phenomena, allowing relative ease of access to dynamical observables via localized measurements of photon fields.

In this work, we investigate the time evolution of nonlinear arrays, going beyond the one- or two-photon limit into a strongly correlated many-body regime. We find evidence for an interaction-induced 'freezing' of domain walls of photons in initially half-filled one-dimensional array systems. The resonator nonlinearity must be sufficiently large for localization effects to set in, beyond which the photon population remains trapped in half of the system. We show that a semi-classical (SC) treatment similar to that first presented in [11] for the limiting case of two coupled resonators predicts a sharp transition between localized domain formation and delocalized dynamics in which photons tunnel between both halves of the system.

Going beyond the SC approach, fully quantum calculations confirm the frozen photon dynamics for strongly nonlinear arrays while also revealing features not present in the SC calculation.

In some ways, our findings can be seen as a photonic analogue of the non-equilibrium dynamics of XXZ chains where it has been recently shown that strong nearest-neighbour interactions can lead to the formation of polarized domains which strongly influence the transport properties of spin chains [15]. These ferromagnetic domains have been shown to be stable. They are spectrally separated from mobile states of the system which are capable of breaking them.

[4] Authors to whom any correspondence should be addressed.

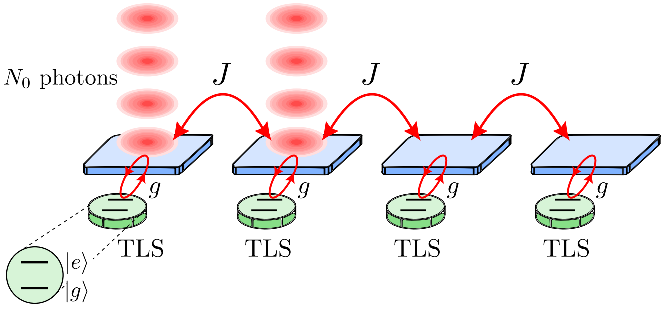

Figure 1. Schematic of our system and the initial conditions considered. The left half of a one-dimensional array with M resonators (here $M = 4$) is initialized in a Fock state of N_0 photons (in this particular schematic, $N_0 = 4$). Each resonator is coherently coupled to its two nearest neighbours with the associated tunnelling rate J. Each resonator is also coherently coupled to a TLS with a Jaynes–Cummings coupling parameter g.

We show a related but distinct photonic equivalent. Our 'domains' of photons remain trapped over timescales larger with respect to characteristic system rates due to a vanishing overlap between the initial pumped states we consider and propagating modes of the system in the limit of large local interactions.

2. The system

The system we consider is a one-dimensional linear array of M coupled optical resonators. Each resonator features a relevant mode of frequency ω_r, and is coherently coupled to its nearest neighbours, as shown schematically in figure 1. A single two-level system (TLS) with the transition frequency ω_a is coherently coupled via a Jaynes–Cummings interaction to each resonator, with coupling strength g. In this work, we consider only the on-resonance case $\Delta = \omega_r - \omega_a = 0$. The governing Hamiltonian is then the well-known Jaynes–Cummings–Hubbard (JCH) Hamiltonian:

$$\hat{\mathcal{H}} = \sum_j \left[\left(\omega_r \hat{a}_j^\dagger \hat{a}_j + \omega_a \hat{\sigma}_j^+ \hat{\sigma}_j^- + g\left(\hat{a}_j^\dagger \hat{\sigma}_j^- + \hat{a}_j \hat{\sigma}_j^+\right)\right]\right. \\ - J \sum_{\langle j,j'\rangle} \hat{a}_j^\dagger \hat{a}_{j'}. \quad (1)$$

Here, $\hat{a}_j$ is the photon destruction operator for the resonator j and $\hat{\sigma}_j^\pm$ are the raising/lowering operators for the TLS coupled to the resonator j. The set of nearest-neighbour resonators is denoted by $\langle j, j'\rangle$. The Hamiltonian $\hat{\mathcal{H}}$ commutes with the total excitation number operator

$$\hat{\mathcal{N}} = \sum_j \left(\hat{a}_j^\dagger \hat{a}_j + \hat{\sigma}_j^+ \hat{\sigma}_j^-\right). \quad (2)$$

We remove the free evolution of the resonators coupled to the TLS by transforming to a frame rotating at $\omega_r = \omega_a$, such that only the competing terms governing the atom–resonator interaction and resonator tunnelling remain. We note that for the form of the Jaynes–Cummings coupling to be valid in equation (1), we must operate in the regime $g \ll \omega_r = \omega_a$. Throughout this work, we consider initial states with one half of the system contiguously populated with N_0 photons per resonator, while the other half remains empty. The TLS in each resonator is initialized in its ground state:

$$|\Psi(0)\rangle = \prod_{j=1}^{M/2} |g, N_0\rangle_j \otimes \prod_{j=M/2+1}^{M} |g, 0\rangle_j, \quad (3)$$

where $|g, n\rangle_j$ denotes a photonic Fock state of n photons in the resonator j.

Generalizing [11] to the case of an extended system, of central interest in the following analysis will be the photon imbalance $Z(t)$ between the left (L) and right (R) halves of the system, as defined by

$$Z(t) = \frac{\sum_{j=1}^{M/2} \langle \hat{a}_j^\dagger \hat{a}_j\rangle(t) - \sum_{j=M/2+1}^{M} \langle \hat{a}_j^\dagger \hat{a}_j\rangle(t)}{\sum_{j=1}^{M} \langle \hat{a}_j^\dagger \hat{a}_j\rangle(t)}. \quad (4)$$

In particular, we find that time-averaging $Z(t)$ neatly encapsulates details of the photon dynamics. We denote such time averages in the following by $\bar{Z}$. Throughout this work, $\bar{Z}$ is defined as the average of $Z(t)$ over the interval $tJ \in [0, 20]$. We have empirically found that this interval is sufficiently long to capture the nature of the long-time dynamics and short enough that the array is in a non-trivial state at the end of the interval once losses are included. Values of $\bar{Z}$ close to zero imply the delocalization of photons across the two halves of the system, either oscillating back and forth in some manner or reaching an approximately even distribution. Meanwhile, $\bar{Z} \approx 1$ implies a photon population trapped in one side of the system for a substantial period of time.

3. Semi-classical treatment

We begin our analysis of the dynamics of the system equation (1) at the SC level, thereby making contact with the related previous work by the authors of [11]. We first use the Heisenberg equation of motion $\frac{\mathrm{d}}{\mathrm{d}t}\hat{O} = \mathrm{i}[\hat{\mathcal{H}}, \hat{O}]$ to generate evolution equations for the photonic and TLS operator expectation values. As the SC approximation entails factorizing the expectation values of the operator products into the products of expectation values (e.g. $\langle \hat{a}_j^\dagger \hat{\sigma}_j^-\rangle = \langle \hat{a}_j^\dagger\rangle\langle \hat{\sigma}_j^-\rangle$), we only need to generate equations for the three operators $\hat{a}_j, \hat{\sigma}_j^-, \hat{\sigma}_j^z$.

Defining $(\alpha_j, m_j, z_j) \equiv (\langle \hat{a}_j\rangle, \langle \hat{\sigma}_j^-\rangle, \langle \hat{\sigma}_j^z\rangle)$, we obtain the set of coupled differential equations for their evolution:

$$\begin{aligned} \dot{\alpha}_j &= -\mathrm{i}\omega_r \alpha_j - \mathrm{i}g m_j + \mathrm{i}J((1-\delta_{j,1})\alpha_{j-1} + (1-\delta_{j,M})\alpha_{j+1}) \\ \dot{m}_j &= -2\mathrm{i}\omega_a m_j + \mathrm{i}g\alpha_j z_j \\ \dot{z}_j &= -2\mathrm{i}g\left(\alpha_j m_j^* - \alpha_j^* m_j\right), \end{aligned} \quad (5)$$

where the delta functions take into account the open boundary conditions.

An SC description is not capable of properly describing the quantum mechanical Fock states of equation (3). We therefore take an array pumped with coherent states with the same photon number $\hat{a}_j^\dagger \hat{a}_j = N_0$ as initial conditions of equations (5):

$$\begin{aligned} \alpha_j &= \begin{cases} \sqrt{N_0} : j \leqslant \frac{M}{2} \\ 0 : j > M/2, \end{cases} \\ z_j &= -1, \\ m_j &= 0. \end{aligned} \quad (6)$$

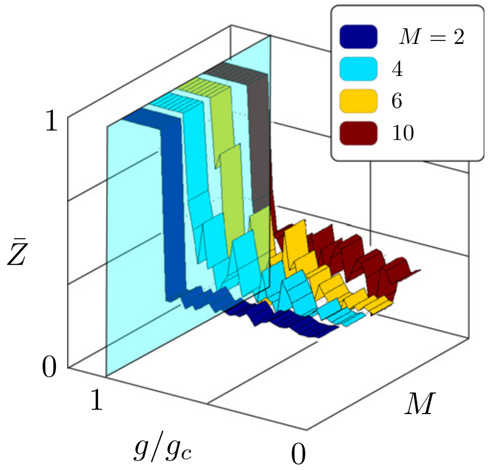

Figure 2. The time-averaged photon imbalance $\bar{Z}$ according to an SC treatment of an M-resonator array. The transparent blue plane marks the critical coupling predicted by SC theory for the case of $M = 2$ resonators [11].

We note that it has been shown that for the case of $M = 2$ resonators, a qualitative change in the population imbalance dynamics occurs sharply at a critical coupling $g_c \approx 2.8\sqrt{N_0}J$ [11]. Specifically, for $g < g_c$, photons move between the resonators with a characteristic tunnelling time. Around $g = g_c$, this period diverges, leading to a 'self-trapped' regime for $g > g_c$ in which the photon population remains localized in one resonator.

With these results in mind, we now look at the time-averaged photon imbalance $\bar{Z}$ for larger arrays as calculated by the time evolution of the set of equations (5), as shown in figure 2. We see that SC theory still predicts a sharp localized/delocalised transition at the critical coupling $g = g_c$, regardless of the system size M.

4. The fully quantum regime

Going now beyond SC theory, which can only be valid in the limit of a large number of excitations, we investigate whether an analogue of the localization predicted by the SC equations persists in the fully quantum regime of a few ($N_0 \leqslant 4$) excitations. Explicitly constructing a matrix representation of the Hamiltonian of equation (1) and time evolving by applying the unitary operator $\mathcal{U}(t) = \exp(-i\mathcal{H}t)$ to the initial state, $|\Psi(t)\rangle$ becomes numerically challenging beyond even $M = 2$ resonators. The two-species nature of the JCH Hamiltonian, coupled with the necessity of retaining a sufficient number of photons per resonator in calculations so as to avoid the truncation error, leading to a large Hilbert space dimension. Some progress is possible by projecting the dynamics into fixed particle number subspaces; however, we turn instead to a compact matrix product state (MPS) representation of the wavefunction [16]. This representation is ideally suited for representing the state of one-dimensional systems with at most nearest-neighbour couplings as in our case. Efficient and accurate Hamiltonian evolution of the MPS is achieved via the time-evolving block decimation (TEBD) algorithm [17].

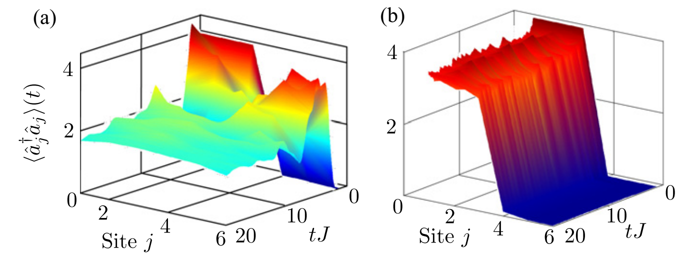

Figure 3. Photon number evolution for an $M = 6$ resonator system with the first three resonators pumped with $N_0 = 4$ photon Fock states at $t = 0$. (a) A weak nonlinearity $g = 0.1J$. (b) Strong Jaynes–Cummings nonlinearity $g = 15J$. The apparent reduction in the mean photon number per site over time is accounted for by the TLS excitation—we have checked that the total excitation in the system is preserved by our numerics to 1% over the simulation interval. Simulation parameters: all calculations kept a minimum of $n_{\max} = \min(N_0M/2, 7)$ photons per resonator in the computational basis. An MPS truncation parameter of $\chi = 100$ was found to be sufficient to avoid cumulative errors.

Figure 3 shows TEBD simulations of the local photon density in an $M = 6$ resonator array with the left half initially pumped with Fock states of $N_0 = 4$ photons, for two Jaynes–Cummings nonlinearities, one weak and one strong (relative to the photon tunnelling rate J). For weak nonlinearities, photons initially oscillate between the two halves of the system, reflecting from the boundaries and eventually leading to a uniformly distributed population, i.e. zero photon imbalance $\bar{Z} \approx 0$.

For arrays with strong nonlinearities, on the other hand, such as shown in figure 3(b), photons remain essentially trapped on the left-hand side of the system for very long times (>98% of the photon population remains in the first three sites over the simulation window). The self-trapping phenomenon predicted by SC theory then seems to persist in the fully quantum regime of few photons.

Figure 4 however characterizes the emergence of these domains of 'frozen' photons, showing that SC theory is insufficient to fully capture all qualitative details of the effect in the low-excitation regime. Figure 4(a) shows the results of rigorous TEBD simulations for arrays of different sizes, pumped with different numbers of initial photons. We see that the 'transition' between delocalized ($\bar{Z} \approx 0$) and localized ($\bar{Z} \approx 1$) dynamics becomes broader with increasing system size M, and never resembles the sharp SC transition of figure 2. The qualitative trend, however, towards localization with increasing nonlinearity g occurs irrespective of M. Figure 4(b) meanwhile shows that for $N_0 > 1$, the magnitude of the initial excitation does not significantly affect the rate at which the system approaches the 'frozen' regime. Interestingly, however, we find that the case of a single photon pumped into the left half of the system never exhibits localization behaviour, no matter how large is the Jaynes–Cummings nonlinearity g.

We can understand this qualitative difference in behaviour between $N_0 = 1$ and $N_0 > 1$ by examining the initial state $|\Psi(0)\rangle$ in the eigenbasis of the Hamiltonian $\hat{\mathcal{H}}$. We work in the Hilbert subspace spanned by eigenvectors $\{\Psi_j\}$ commuting with the total excitation operator $\hat{\mathcal{N}}$ with the eigenvalue $n = N_0M/2$ and calculate the overlap of the initial state with

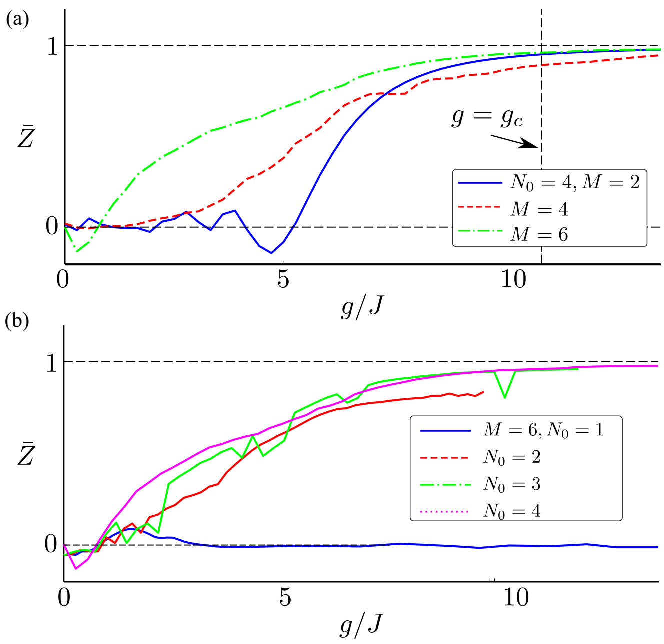

Figure 4. An exploration of the time-averaged photon imbalance $\bar{Z}$. (a) As a function of the system size M, holding the initial excitation N_0 fixed. (b) $\bar{Z}$ as a function of the initial excitation N_0, holding instead the system size fixed.

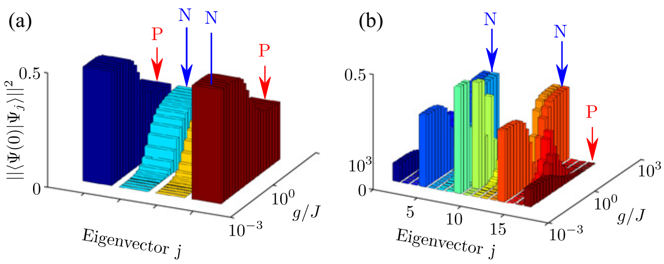

Figure 5. The projection of the initial state $|\Psi(t = 0)\rangle$ into each of the eigenstates spanning the subspace consistent with the total number of excitations in the system, $N_0M/2$, for the simplest case of a dimer of $M = 2$ resonators. (a) Overlaps for an initial excitation of just $N_0 = 1$ photon in the first resonator. (b) The corresponding projections for an initial state of $N_0 = 4$ photons. Both plots show the evolution of the different projections as the Jaynes–Cummings nonlinearity g is ramped up. Eigenmodes marked 'P' (for 'propagating') have a nonzero photon correlation function C across the two halves of the system. Modes marked 'N' have vanishing C as g increases. Note that the larger dimension of the subspace for $N_0 = 4$ results in more eigenstates in (b).

each of these eigenstates $||\langle\Psi(0)|\Psi_j\rangle||^2$. These overlaps are presented in figure 5 for a minimal 'array' of $M = 2$ resonators with both $N_0 = 1$ and $N_0 = 4$ initial photons, as a function of the nonlinearity g. As expected, we see a shuffling of excitation between various eigenmodes as g changes. The properties of the eigenmodes having significant overlap with the initial state for a given nonlinearity g determine the properties of the time evolution of the system. Of particular relevance is a measure of the 'photon current' through the centre of the system, or alternatively the degree of photon delocalization across the halves of the array. Both these quantities are reflected in a finite value of the expectation value $C = |\langle\hat{a}^{\dagger}_{M/2}\hat{a}_{M/2+1}\rangle|$. On measuring C for each of the modes $|\Psi_j\rangle$, we find that the initial

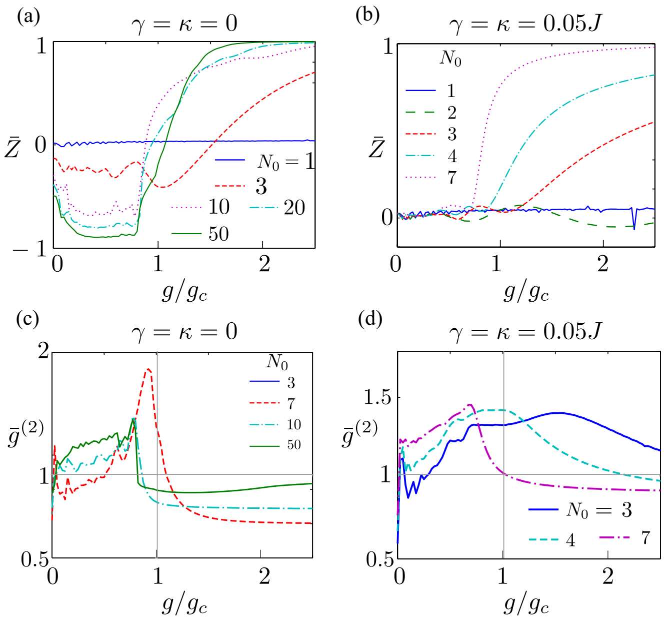

Figure 6. Charting the localization–delocalization transition for a minimal $M = 2$ resonator array through both the time-averaged photon imbalance $\bar{Z}$ (top row) and the time-averaged photon correlator $\bar{g}^{(2)}$ (bottom row) for increasing initial photon number N_0. $\bar{Z}$ and $\bar{g}^{(2)}$ are calculated for systems with no losses (left column) and for small finite loss rates $\gamma = \kappa = 0.05J$.

state for the case of a single pumped initial photon $N_0 = 1$ has a finite overlap with 'current-carrying' modes even in the limit of a large nonlinearity g. In contrast, only non-propagating modes are substantially excited for larger excitations $N_0 > 1$, leading to the frozen domains of figure 3(b).

5. Probing the frozen dynamics in an experiment

Finally, we present calculations showing experimentally relevant photonic observables which give signatures of the transition between the localized and delocalized dynamics. While the photon number imbalance $\bar{Z}$ between the two halves of the system may be measurable via quantum non-demolition measurements on frequency shifts of the TLS, we focus here on purely photonic observables that can be extracted from the emitted photons from the structure. In particular, we find that the measurement of the local second-order photon correlations $g^{(2)}_L = \langle\hat{a}^{\dagger}_j\hat{a}^{\dagger}_j\hat{a}_j\hat{a}_j\rangle/\langle\hat{a}^{\dagger}_j\hat{a}_j\rangle^2$ yields signatures of the transition. Figure 6 shows that the freezing of population in one-half of the system (where the photon imbalance is $\bar{Z} \approx 1$) is accompanied by a qualitative change in the on-site correlations from $g^{(2)} > 1$ to $g^{(2)} \approx 1$. Figures 6(a) and (c) show that both the photon imbalance and the time-averaged correlator $g^{(2)}$ approach a limiting behaviour as the initial number of photons grows large, with a sharp transition in observables in the vicinity of the critical point predicted by SC theory $g = g_c$.

Experimentally, the photon statistics encoded in $g^{(2)}$ are mapped on to the statistics of photons leaking from resonators with finite line widths as characterized by a loss rate γ. To assess whether the correlator $g^{(2)}$ can serve as a probe of the transition in realistic settings with a finite resonator loss rate, we include Markovian photon loss processes at rate γ and TLS de-excitation at rate κ via a quantum master equation formalism, time evolving the system density matrix ρ from the initial state of equation (3) under the evolution:

$$\dot{\rho}(t) = -\mathrm{i}[H, \rho] + \sum_{i=L,R} (\gamma \mathcal{L}[a_i] + \kappa \mathcal{L}[\sigma_i^-]), \qquad (7)$$

where the action of the dissipator $\mathcal{L}$ is defined as $\mathcal{L} = (2O\rho O^\dagger - O^\dagger O\rho - \rho O^\dagger O)/2$. Figures 6(b) and (d) show that the introduction of a finite loss rate acts to smear the transition, pushing localization to larger nonlinearities g. However, for sufficiently large initial photon pumping (around $N_0 \approx 7$), the statistics of the emitted photons can be used to infer a change from delocalized physics (characterized by $\bar{g}^{(2)} > 1$) to the localized case ($\bar{g}^{(2)} \approx 1$). The value of gamma we use leads to a maximum ratio $g/\gamma \approx 200$, within reach of near future experiments in circuit QED architectures [18].

6. Conclusions

We have demonstrated the existence of a novel strongly correlated regime of 'frozen' photons in optical resonator arrays with large Jaynes–Cummings-type nonlinearities. For a sufficiently large initial excitation of part of the resonator array, the photon dynamics are dramatically suppressed due to a very small overlap with propagating modes of the system. As little as two pumped photons per resonator are sufficient to observe signatures of 'frozen' domains of photons, allowing access to the truly quantum few-excitation regime.

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