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SIMULATING TOPOLOGICAL EFFECTS WITH PHOTONS IN COUPLED QED CAVITY ARRAYS

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We provide a pedagogical account of an early proposal realizing fractional quantum Hall effect (FQHE) using coupled quantum electrodynamics (QED) cavity arrays (CQCAs). We start with a brief introduction on the basics of quantum Hall effects and then review the early proposals in the simulation of spin-models and fractional quantum Hall (FQH) physics with photons in coupled atom-cavity arrays. We calculate the energy gap and the overlap between the ground state of the system and the corresponding Laughlin wavefunction to analyze the FQH physics arising in the system and discuss possibilities to reach the ground state using adiabatic methods used in Cavity QED.

Keywords: Coupled quantum electrodynamics cavity arrays; fractional quantum Hall effect.

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1. Introduction to Quantum Simulations with Coupled Cavity Arrays

Quantum simulators offer a promising alternative when analytical and numerical methods fail in analyzing models with strong correlations characterizing condensed matter systems.[1] The most famous example of this kind, with numerous applications in describing effects such as quantum phase transitions, is the Hubbard model. The latter has been simulated using ion traps,[2] but found its optimal realization with cold atoms in optical lattices.[3,4] More recently, motivated by great progress in the field of Cavity quantum electro-dynamics (QED) and quantum nonlinear optics, proposals to use strongly correlated photons as quantum simulators going beyond linear optics[5] have appeared.[6,7] Initially, it was shown that one could generate strongly correlated states of photons with an array of coupled cavities doped with real or artificial atoms. The photons, although normally noninteracting, are made to interact through the doped atoms and thereby emulate many body effects such as Mott to Superfluid transition.[8–10]

The basic mechanism behind coupled cavities is evanescent coupling. The light mode of interest is strongly confined in a cavity and the Wannier mode decays exponentially outside the cavity. The overlap of the modes between two adjacent cavities allows photons to hop between the cavities, while strong confinement suppresses direct hopping to non-nearest-neighbor cavities. The resulting dynamics is described by the tight-binding model for photons, analogous to the model used in condensed matter physics to describe electrons moving in a lattice of ions and to the system of atoms moving in an optical lattice. Introducing atoms to the cavities create interactions between photons, resulting in what is known as photon blockade which was used to mimic the repulsive interactions necessary to emulate strongly interacting models. The resulting Jaynes–Cummings–Hubbard Hamiltonian was shown to be analogous to the Bose–Hubbard model in many ways but also to exhibit richer structure due to the hybrid light-matter nature of the excitations in question. The ability to address individual sites and perform well-developed quantum optical measurements provided a strong motivation to study and develop coupled QED cavity arrays (CQCAs) as an emulator of condensed matter systems.

In the early photonic quantum phase transition works mentioned above, the XY spin model emerged naturally because, in the Mott regime, two photons cannot occupy the same cavity mode and the Hilbert space dimension of a single site is thus reduced to two, mimicking the spin-1/2 system.[10] Photonic analogues of more complex spin systems in coupled QED cavities soon followed where the introduction of more than one atoms per cavity was shown to generalize the setup to high-spin Heisenberg models.[11] Exotic models such as the Kitaev hexagonal lattice can also be implemented exploiting polarized photons.[12] Recently, one of us has shown with collaborators that an effective gauge field for photons can be introduced in coupled QED cavities by introducing site-dependent lasers.[13] For a two-dimensional (2D) CQCA, this means that an effective magnetic field for photons is induced. We would like to mention here a number of more recent proposals and experiments employing photons for topological effects in a number of different platforms ranging from driven optical setups[14,15] to micro-wave Circuit QED, photonics and metamaterials.[16–20]

In this paper, we review some of the basics of the early CQCA array proposals and focus on the effective gauge fields implementations and the Fractional Hall effect work. Pedagogical introduction to integer and fractional quantum hall effects

(FQHEs) are given first with a brief remark on topological effects arising in these systems. Explanations on how to realize a spin model in coupled QED cavities and their relevance to (FQH) systems is provided next, followed by calculations showing some relevant quantities that exhibit the FQHE arising in the engineered CQCA.

2. Quantum Hall Effect Basics Revisited

Quantum Hall effect is a phenomenon occurring in a 2D electron gas under a strong magnetic field in the transverse direction.[21] When a current is induced in such a system the Hall current develops in the perpendicular direction as in the classical Hall configuration. In the quantum case, however, the Hall resistance exhibits plateaus at precisely defined values, i.e., the resistance does not change as one varies the magnetic field, for example. When the plateau occurs at an integer multiple of h/e^2 the effect is called the integer quantum Hall effect and can be understood in terms of single particle eigenstates called Landau levels. Somewhat surprisingly, however, the robustness of the quantization of the Hall resistance cannot be explained without the impurities providing the mobility gap. Such robustness has been explained in terms of the topological nature of the geometric phase induced by the gauge field.[22]

The plateaus also occur at rational fractions of h/e^2, in which case the effect is called the FQHE.[23] The FQHE results from strong interactions between the electrons and can be understood in terms of fractional charges that have fractional statistics. These quasiparticle excitations are called anyons and form the basis of topological quantum computation.

2.1. *Integer quantum Hall effect*

The phenomena of quantized hall resistance can be readily understood in terms of Landau levels as alluded to earlier. Here, we give a brief summary and refer the reader to excellent lecture notes by Girvin[24] for a more detailed account.

The Hamiltonian for 2D noninteracting gas of electrons under perpendicular magnetic field is given by

$$H = \frac{1}{2m}(p + eA)^2 \,, \tag{1}$$

where $-e$ is the charge of the electron. It is convenient to choose the Landau gauge, $A_x = -By$ and $A_y = 0$, in which case the Hamiltonian is independent of x and the solutions can be written as $\psi_k(x, y) = e^{ikx} f_k(y)$. The effective one-dimensional (1D) Hamiltonian acting on this function is

$$H_k = \frac{1}{2m}p_y^2 + \frac{1}{2}m\omega_c^2(y - kl_B^2)^2 \,, \tag{2}$$

where $\omega_c = eB/m$ is the cyclotron frequency and $l_B = \sqrt{\hbar/eB}$ is called the magnetic length. The electrons therefore occupy discrete harmonic oscillator levels named Landau levels. In the presence of a large magnetic field, each Landau level has a huge degeneracy due to the freedom in choosing the y momentum k. It turns out that there is one state per Landau level per flux quantum, $\Phi_0 = h/e$, i.e., the total number of states in each Landau level is $N_\phi = eBA/h$, where A is the area of the sample.

Naively, therefore, the quantized plateaus can be explained as follows. With the changing magnetic field strength, the degeneracy of the Landau levels change, and when the Fermi energy lies between the Landau levels the number of participating electrons in the Hall current is fixed, resulting in a quantized value of Hall resistance while the magnetic field is swept within the gap. The quantized Hall resistance then depends on the filling factor $\nu \equiv N/N_\phi = n$, where n is an integer, according to the argument just provided. This argument suffers a slight problem, however, in that it is not possible for the Fermi energy to lie between the Landau levels while sweeping the magnetic field strength. What gives the experimental stability of the quantized hall conductance is, somewhat counter-intuitively, disorder. In the presence of disorder, the Landau levels broaden and form bands of extended and localized states. When the Fermi energy varies within a band of localized states called the mobility gap, the transport property is unaffected and a plateau of finite width can be observed. Disorder plays a very important role in giving the robustness to the quantum Hall effect.

2.2. *Fractional quantum Hall effect*

The FQHE generally occurs within the lowest Landau level, the earliest example found being $\nu = 1/3$. Naively, one would expect the Hall plateaus to be absent in this case, because there are still unfilled states with the same energy. What happens, of course, is that the degeneracy is broken by the coulomb interaction between the electrons. Electrons are highly correlated due to the interaction and the ground state becomes nontrivial, whose form is hard to obtain from a perturbative analysis due to the large degeneracy. It was not long, however, before an inspirational guesswork by Laughlin solved the problem.[25] The strongly correlated electronic state in the FQHE was found to be excellently described by Laughlin's variational wavefunction, which in the symmetric gauge ($A_x = -By/2$ and $A_y = Bx/2$) reads

$$\Psi_q(z_j, z_j^*) = \prod_{k<l} (z_k - z_l)^q e^{-\sum_j {z_j}^2/4}\,, \tag{3}$$

where $q = 1/\nu$ and $z_j = x_j + iy_j$ is a complex variable representing the position of the jth electron in units of the magnetic length l_B. Quasi-excitations in such systems are separated from the ground state by an energy gap and can be created by introducing a flux quantum through a thin solenoid. Interestingly, such quasi-excitations carry with them a fractional charge. This has lead to development of many useful pictures explaining various aspects of FQHE such as composite fermions and hierarchy picture.[26,27]

2.3. *Topological effects in quantum Hall systems*

The robustness of quantized Hall resistance has topological origin as first explained by Laughlin,[22] where the author showed that the gauge invariance is at the root of the robustness. Soon after, the influential work by Thouless, Kohmoto, Nightingale and den Nijs (often known as TKNN) showed that the Hall conductance calculated from the Kubo formula must be quantized due to single-valuedness of the wave-function.[28] They have considered the Hall system in a lattice and concluded that the Hall conductance in such a system is proportional to an integral of a certain quantity over the first (magnetic) Brillouin Zone, summed over the filled bands and that this integral must be an integer multiple of some factor. Simon showed that this integer is related to the so-called (first) Chern number, related to the Berry phase.[29]

The topological argument above applies to both the integer and FQHEs and therefore does not exhibit the principal differences between them. The main difference between them is that the FQHE arises from interacting particles. A stark manifestation of the collective many body nature is the anyon, a particle with neither Bosonic nor Fermionic but “any” statistics,[30,31] that arises as the quasiparticle in FQH systems.[32] In some FQH systems, notably for $\nu = 5/2$, the quasiparticle excitations are non-Abelian anyons which has potential use in topological quantum computation.[33]

3. From Spin Models to Fractional Quantum Hall Effect with Photons in Coupled QED Cavities

For photons (or bosons), there is no direct analogue of quantized plateaus because photons do not fill up the states up to the Fermi energy, i.e., there is no Pauli principle for photons.[34] The Laughlin state is still a good (sometimes exact) variational ground state for bosons, however, with an even-denominator filling factor. $\nu = 1/2$ is the analogue of the usual $\nu = 1/3$ FQH state.[35] This is because the Laughlin state is the unique state that minimizes the repulsive short-range potential within the lowest Landau level. It can be justified in the usual sense of justifying the Laughlin state, except that the inverse of the filling factor has to be an integer to account for the bosonic nature of the particles. The kinetic plus the magnetic part of the Hamiltonian is automatically minimized by using the degenerate wave-functions within the lowest Landau level, while the short-range bosonic interaction term is minimized (set to zero in fact), because the Laughlin state is zero whenever two particles coincide.

We discuss the first proposal to realize FQH state in a CQCA[13] in detail here. We will first explain how to generate a spin Hamiltonian where the effective spin states are represented by hyperfine atomic levels in each of the resonators. The proposal realizes the Laughlin state by employing an effective 2D spin model where the spin exchange is mediated by intercavity hopping of virtually excited cavity photons.

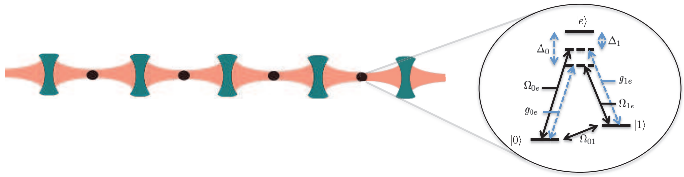

Fig. 1. 1D CQCA doped with single atoms for realization of Heisenberg type spin Hamiltonians.

3.1. *Spin models in coupled QED cavities*

Assume cavities arranged in the form of a lattice (typically, we consider a regular lattice such as a 1D chain or 2D plane). Each cavity is doped with a single three-level system (which we refer to as an atom) and two ground levels are used to represent an $s = 1/2$ spin (in a rotated basis, as will be seen later). We start by recalling that in terms of two states $|\downarrow\rangle$ and $|\uparrow\rangle$ of one atom, the spin-1/2 system can be described in terms of operators $s^Z = 1/2(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|)$, $s^+ = |\uparrow\rangle\langle\downarrow|$ and $s^- = |\downarrow\rangle\langle\uparrow|$.

For simplicity, let us consider a 1D CQCA depicted in Fig. 1. $|0\rangle$ and $|1\rangle$ are assumed to be stable, whereas the excited level $|e\rangle$ is not. Let us denote by $|\psi_j\rangle$ the state $|\psi\rangle$ of the atom in the jth cavity. In the rotating frame, the Hamiltonian describing the system reads

$$\begin{aligned} H = &\sum_j [e^{i\Delta_1 t}\Omega_{0e}\sigma_j^{e0} + e^{i\Delta_0 t}\Omega_{1e}\sigma_j^{e1} + \text{h.c.}] \\ &+ \sum_j [(e^{i\Delta_0 t} g_{0e}\sigma_j^{e0} + e^{i\Delta_1 t} g_{1e}\sigma_j^{e1}) a_j + \text{h.c.}] \\ &+ \sum_j \left[\frac{\Omega_{01}}{2}(\sigma_j^{01} + \sigma_j^{10}) + J(a_j^\dagger a_{j+1} + a_j a_{j+1}^\dagger)\right], \end{aligned} \tag{4}$$

where $\sigma_j^{xy} = (|x\rangle\langle y|)_j$ $(x, y = 1, 0, e)$; a_j is the annihilation operator for the jth cavity mode; Ω_{xy} is the Rabi frequency of the classical field driving the transition $|x\rangle \leftrightarrow |y\rangle$, g_{xy} is the corresponding atom-cavity coupling rate and J is the inter-cavity hopping rate of photons. Note that both the transitions are coupled to the same cavity mode and the Raman transitions are completed by appropriate classical fields. The transition between $|0\rangle$ and $|1\rangle$ can be induced, for example, by a two-photon process.

Working in a rotated spin basis such that $|\uparrow\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle)$ and $|\downarrow\rangle = (1/\sqrt{2})(|0\rangle - |1\rangle)$, it can be shown that the above Hamiltonian transforms to[11]:

$$\begin{aligned} H = &\sum_{j=1}^N [K_3(S_j)^2 + K_4(S_j^Z)^2 + BS_j^Z] \\ &+ \sum_{j=1}^N [K_1(S_j^X S_{j+1}^X + S_j^Y S_{j+1}^Y) + K_2 S_j^Z S_{j+1}^Z], \end{aligned} \tag{5}$$

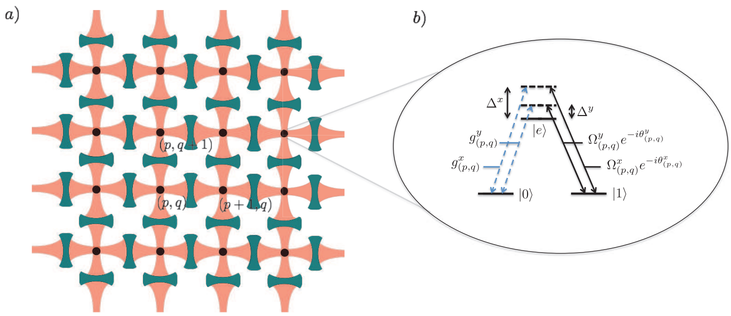

Fig. 2. (a) 2D CQCA realizing the FQHE. Each site labeled by two position indices (p, q) comprises of two orthogonal cavity modes and a three-level atom. The cavity modes mediate the hopping between adjacent sites. (b) Atomic level scheme. The cavity mode x (y) is detuned by Δ^x (Δ^y) and is coupled to the atomic transition $|e\rangle\langle 0|$ with coupling strengths $g^x_{(p,q)}$ $(g^y_{(p,q)})$. The transition $|e\rangle\langle 1|$ is driven by the laser fields with Rabi frequencies $\Omega^x_{(p,q)}e^{-i\theta^x_{(p,q)}}$ and $\Omega^x_{(p,q)}e^{-i\theta^x_{(p,q)}}$. The subscripts denote the position of the site.

where K_j and B are analytic functions of J, Ω_{01}, g^2_{0e}/Δ_{0e} and $\mu_{i,j} = \mu_i \pm \mu_j$ with $\mu_j = g_{je}\Omega^*_{je}/\Delta_{je}$. Note that two more laser fields have to be brought in (not shown in the figure) to achieve full control of the individual parameters. In this case all terms can be determined freely, making the model general enough to cover any anisotropic or isotropic Heisenberg spin chains.[11]

If $K_2 = K_3 = K_4 = B = 0$, the Hamiltonian becomes of the type $\sum_j(\sigma^+_{j+1}\sigma^-_j + \text{h.c.})$. This can be thought of as strongly interacting bosons hopping between the sites such that no two bosons can occupy the same site. We want to discuss FQHE arising in such a system, for which another spatial dimension and a synthetic magnetic field has to be introduced.

3.2. *Fractional quantum Hall states of photons*

To realize the FQH states of photons, the above spin system has to be generalized to 2D coupled QED cavities. The system is depicted in Fig. 2. Here, the atomic scheme is a little simpler than the previous one, except for the fact that there are now two cavity modes interacting with an atom. The atoms now interact with two independent cavity modes in the x and y directions and the cavity modes are coupled only to the atomic transition between the ground state $|0\rangle$ and the excited state $|e\rangle$ with the coupling strength $g^\mu \equiv g^\mu_{0e}$. They are detuned by Δ^μ, where μ denotes the x or y mode. The classical control fields of Rabi frequencies $\Omega^\mu e^{-i\theta^\mu}$ complete the Raman transitions between the ground states $|0\rangle$ and $|1\rangle$ with zero two-photon detuning. The total Hamiltonian reads

$$H = \sum_{\mu=x,y}\sum_{j=(p,q)} g^\mu e^{-i\Delta^\mu t}a^\mu_j(|e\rangle\langle 0|)_j + \Omega^\mu e^{-i\theta^\mu_j}e^{-i\Delta^\mu t}(|e\rangle\langle 1|)_j + \text{h.c.}$$

$$- \sum_{p,q}(J^x a^{x\dagger}_{p+1,q}a^x_{p,q} + J^y a^{y\dagger}_{p,q+1}a^y_{p,q}) + \text{h.c.}\,, \tag{6}$$

where J^x and J^y are the intercavity hopping rate in the x and y directions and (p, q) denotes the site on the qth row and the pth column. We assume $\Delta^\mu \gg g^\mu \gg \Omega^\mu, J^\mu$ to suppress the cavity photon and to ensure that the interaction is very strong. We also assume $\Delta^x - \Delta_y \gg g^x, g^y$ to prevent cross-coupling between the two cavity modes. Adiabatic elimination of the excited state then yields the effective Hamiltonian

$$\begin{aligned} H = &\sum_{\mu=x,y} \sum_{j=(p,q)} \delta^\mu a_j^{\mu\dagger} a_j^\mu (|0\rangle\langle 0|)_j + \omega_\mu (e^{i\theta_j^\mu} a_j^\mu \sigma_j^+ + \text{h.c.}) \\ &- \sum_{p,q} (J^x a_{p+1,q}^{x\dagger} a_{p,q}^x + J^y a_{p,q+1}^{y\dagger} a_{p,q}^y) + \text{h.c}\,, \end{aligned} \tag{7}$$

where $\delta_\mu = (g^\mu)^2/\Delta^\mu$, $\omega_\mu = g^\mu \Omega^\mu/\Delta^\mu$ and $\sigma_+ = |1\rangle\langle 0|$. In order to reach the desired spin model, we further assume $\delta^\mu \gg J^\mu \gg \omega^\mu$, which can be satisfied if $g^\mu \Delta^\mu \gg J^\mu/g^\mu \gg \Omega^\mu/\Delta^\mu$ holds. This condition suppresses the cavity photons further and the effective Hamiltonian becomes

$$H = -t \sum_{p,q} \sigma_{p+1,q}^+ \sigma_{p,q}^- e^{i(\theta_{p+1,q}^x - \theta_{p,q}^x)} + \sigma_{p,q+1}^+ \sigma_{p,q}^- e^{i(\theta_{p,q+1}^y - \theta_{p,q}^y)} + \text{h.c}\,, \tag{8}$$

where we have assumed the parameters are such that $t = J^x(\omega^x/\delta^x)^2 = J^y(\omega^y/\delta^y)^2$. The above Hamiltonian can be obtained from the Magnus expansion by projecting on the subspace with no cavity photons.

Note that this Hamiltonian is equivalent to

$$\begin{aligned} H = &-t \sum_{p,q} b_{p+1,q}^\dagger b_{p,q} e^{-i\pi\alpha q} + b_{p,q+1}^\dagger b_{p,q} e^{i\pi\alpha p} + \text{h.c} \\ &+ U \sum_{p,q} b_{p,q}^\dagger b_{p,q}^\dagger b_{p,q} b_{p,q}\,, \end{aligned} \tag{9}$$

in the limit $U \gg t$, if we take $\theta_{p,q}^x = -pq\pi\alpha$ and $\theta_{p,q}^y = pq\pi\alpha$. $\alpha = Ba^2/\Phi_0$, where a is the lattice spacing and α is the number of magnetic flux quanta through a lattice cell. The above Hamiltonian describes a system of bosonic particles moving in a 2D square lattice of spacing a in the presence of a magnetic field $B\hat{z}$. It is obtained from

$$H_0 = -t \sum_{\langle j,k\rangle} b_j^\dagger b_k e^{-i\frac{2\pi}{\Phi_0}\int_j^k \mathbf{A}(\mathbf{r})} \cdot d\mathbf{l}\,, \tag{10}$$

describing free bosons in a single Bloch band in the symmetric gauge. Because of the similarity in the Hamiltonian, the Laughlin state in Eq. (3) is expected to be a good ground state wavefunction for the photonic system in the thermodynamics limit, if $\alpha \ll 1$ and $m = 2, 4, 6, \ldots$.

In the finite-sized calculations, however, it is more advantageous to eliminate the edge effects by taking the periodic boundary condition. The solution for the periodic boundary condition in the Landau gauge ($A = (-By, 0)$) has already been found.[36,37] It can be written as

$$\Psi(z_1, z_2, \ldots, z_N) \propto f_{\text{rel}}(z_1, z_2, \ldots, z_N) F_{CM}(Z) e^{\sum_i y_i^2/2l_B}\,, \tag{11}$$

where N denotes the total number of bosons. For a rectangular system of size $L_1 \times L_2$, the relative wavefunction

$$f_{\mathrm{rel}} = \prod_{i<j} [\theta_1((z_i - z_j)/L_1)|\tau]^q \,, \tag{12}$$

with $\tau = iL_2/L_1$ and the center of mass wavefunction

$$F_{\mathrm{CM}}(Z) = \theta \begin{bmatrix} l/q + (N_\phi - 1)/2q \\ -(N_\phi - 1)/2 \end{bmatrix} (qZ/L_1|q\tau) \,, \tag{13}$$

with $l = 0, 1$ labeling the degeneracy due to the ambiguity in choosing the center of mass.[36,38] N_ϕ is the total number of magnetic flux through the entire lattice.

$$\theta \begin{bmatrix} a \\ b \end{bmatrix} (z|\tau) = \sum_n \exp[i\pi\tau(n+a)^2 + 2\pi i(n+a)(n+b)] \,, \tag{14}$$

is the elliptic theta function and θ_1 is defined as the elliptic theta function with $a = b = 1/2$.

Taking the set of parameters $\Delta^\mu/1000 = g^\mu/100 = \Omega^\mu = J^\mu$, corresponding to $\delta^\mu = 100\omega^\mu = J^\mu$, and taking the 4 by 4 lattice with $\alpha = 1/4$ and two bosons such that the filling factor $\nu = 1/2$, the numerically found ground state has the fidelity of 0.976 with the periodic Laughlin state (11). The fidelity of the ground state of the ideal Hamiltonian (9) is $F_{\mathrm{ideal}} = 0.989$ and the fidelity obtained from the photonic system approaches this value as δ^μ/J^μ and J^μ/ω^μ increase. F_{ideal} approaches 1 as $\alpha \to 0$, i.e., in the continuum limit.[39,40] Note that the Landau gauge requires $\theta^y_{p,q} = 0$ and $\theta^x_{p,q} = -pq2\pi\alpha$.

For the filling factor of 1/2, there are two degenerate ground states in the periodic boundary condition. In the lattice version considered in this paper, this degeneracy may or may not be exact depending on the dimension of the lattice. The two lowest ground states, however, are often quite close to each other and are separated from the lowest excited state by a gap larger than the difference between them. In this case, the two ground states ψ_1 and ψ_2 are well-approximated by the periodic Laughlin function as described above. Table 1 gives the values of the energy gap

Table 1. Energy gap and the overlap for different values of α.

	α	ΔE	Overlap		α	ΔE	Overlap
ψ_1	0.333	0.211	0.983	ψ_1	0.143	0.180	0.997
ψ_2	0.333	0.197	0.983	ψ_2	0.143	0.180	0.997
ψ_1	0.25	0.276	0.989	ψ_1	0.125	0.142	0.995
ψ_2	0.25	0.276	0.989	ψ_2	0.125	0.142	0.995
ψ_1	0.2	0.270	0.989	ψ_1	0.111	0.112	0.999
ψ_2	0.2	0.269	0.990	ψ_2	0.111	0.112	0.999
ψ_1	0.167	0.234	0.993	ψ_1	0.1	0.088	0.999
ψ_2	0.167	0.216	0.995	ψ_2	0.1	0.088	0.999

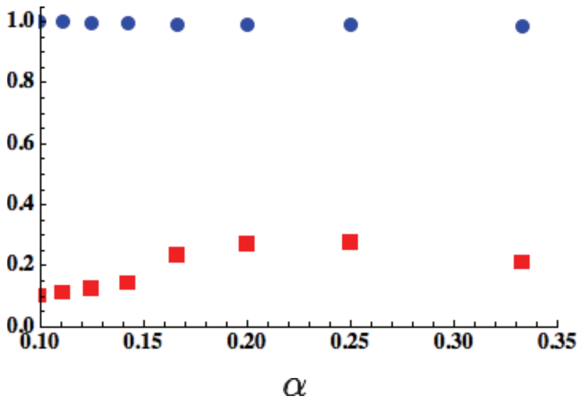

Fig. 3. (Color online) Energy gap (red squares) and the overlap (blue discs) as α is varied.

ΔE to the third lowest energy state and the overlap with the appropriate periodic Laughlin function, as a function of α, the magnetic flux through a unit cell. They are also shown in Fig. 3.

3.2.1. *Adiabatic preparation of the ground state*

Experimentally, it is difficult to impose the periodic boundary condition and one would revert to the normal hard wall boundary condition. In this case, the overlap with the periodic Laughlin function cannot be directly measured, but other predictions of the FQH system should be checked, e.g., incompressibility, correlations between particles, or quasihole excitation spectrum. For this purpose, one should prepare the ground state of the system first, to which we turn our attention here. The ground state (note that the ground state for hard wall boundary condition is unique) can be prepared adiabatically as follows. (i) Prepare all atoms in the ground state $|0\rangle$ by optical pumping or other method with the laser fields turned off. (ii) Select two sites j_0 and j_1 and prepare the atoms in the state $|1\rangle$. (iii) Lower the energy of the state $|1\rangle$ by ϵ at those sites, by creating ac Stark shifts. For example, this can be done easily by shining two lasers on the sites j_0 and j_1. (iv) Gradually increase the Rabi frequencies Ω^x and Ω^y to obtain the desired value of t. (v) Adiabatically decrease the energy shift ϵ to 0. The initial two quanta state was the ground state with finite ϵ and therefore with sufficiently slow turning off of ϵ, the state should have stayed in the ground state of the total Hamiltonian, which for $\epsilon = 0$ is sought after generalized Laughlin state.

For a large enough system, the edge effects should be negligible and the prepared ground state should exhibit characteristic behaviors of FQH states such as incompressibility. Furthermore, one can control the local phases to induce quasiparticles and detect their presence through correlation measurements, for example, we plan to investigate such effects in a future work.

4. Conclusion

We have given a brief pedagogical introduction to integer and FQHEs and discussed in more detail how such a system can be realized in a CQCA system composed of strongly interacting bosons (photons). The energy gap and the overlap with the periodic Laughlin state were calculated for various values of α, the flux through a unit cell, showing that the ground state in such a system indeed shows FQHE. Lastly, a brief description of possible experimental preparation of the ground state was given.

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