Mimicking Interacting Relativistic Theories with Stationary Pulses of Light

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One of the most well known relativistic field theory models is the Thirring model. Its realization can demonstrate the famous prediction for the renormalization of mass due to interactions. However, experimental verification of the latter requires complex accelerator experiments whereas analytical solutions of the model can be extremely cumbersome to obtain. In this work, following Feynman's original proposal, we propose an alternative quantum system as a simulator of the Thirring model dynamics. Here, the relativistic particles are mimicked, counterintuitively, by polarized photons in a quantum nonlinear medium. We show that the entire set of regimes of the Thirring model—bosonic or fermionic, and massless or massive—can be faithfully reproduced using coherent light trapping techniques. The correlation functions of the model can be extracted by simple probing of the coherence functions of the output light using standard optical techniques.

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Introduction.—The Thirring model (TM) is the simplest relativistic quantum field theory that can describe the self-interaction of a Dirac field in $1+1$ dimensions [1]. Various techniques have been developed to predict its correlations function behavior with conclusive results only for the massless fermionic case [2,3]. For the opposite case of fermions with mass, approximations involving the form factors approach have been developed for the n-point correlations whereas a full analytical solution still evades the efforts of seminal theoretical physicists [4–8]. The theory of the bosonic regime and the correlation function behavior is less developed with efforts mainly concentrated in the solving the classical case [9,10].

The possibility to physically simulate the Thirring model dynamics in a different controllable quantum system is thus extremely interesting. An ideal quantum simulation should be capable of reproducing on demand the bosonic or the fermionic cases with tunable interactions and mass terms. In addition direct access to the correlation function behavior will be an advantage. Such a system would not only shed light in the unknown parts of the Thirring model itself, but motivate works for probing of the behavior of other relativistic quantum field theories which are beyond the realm of numerical and experimental techniques. In this work we describe exactly such a setup where the TM is generated counterintuitively in a strongly nonlinear optical system. Here polarized photons are mimicking the behavior of the relativistic fermions where interactions and the mass terms are generated and controlled using quantum slow light techniques.

Recent works in quantum simulations of relativistic theories focused on free fermion simulations using cold atoms and ion systems [11–13]. An interacting fields proposal is described in a recent seminal work where fermions on an optical lattice are used [14] and, more recently, a connection between a variational simulation of fields to a generic cavity QED setup was made [15]. The present work differs in many aspects to all previous quantum simulations. First and foremost it is not based on cold atoms or particles with mass in general, but on a fundamentally different and somewhat counterintuitive approach using photons in a nonlinear optical medium. In addition, our optical setup, being a natural continuum system, has the ability to reproduce the continuous Thirring model directly without resorting to a lattice approach. Finally, both the bosonic and the fermionic cases can be generated by controlling the relevant optical interactions, and in both massless and massive regimes. In our proposal the relativistic dynamics (linear dispersion) and the necessary strong interactions are generated and controlled by employing electromagnetically induced transparency (EIT) techniques [16–18]. The scalings of the correlation functions of the TM, for any regime of interactions, could be simply probed by analyzing the quantum optical coherence functions of the outgoing photons as they exit the medium.

We start our analysis by showing the possibility to generate a nonlinear Dirac type of Hamiltonian using stationary polarized pulses of light, also known as "dark state polaritons"—coherent superpositions of photons in the electromagnetic field and spin wave excitations of the atoms. We then analyze the parameter regime to tune the system to a hard-core regime reproducing the FTM dynamics and continue by showing that both the massless and massive cases are reproducible. We conclude by discussing how one could probe the scaling of correlation functions for any regime of interactions and mass values using standard quantum optical measurements. In this last part, we calculate analytically, for demonstration purposes, the correlation function behavior for the case of the massless fermionic TM where an exact solution is known. This could be used to gauge or first test our simulator before it is used in the territory of theories without known solutions.

Photons as interacting Dirac fermions.—Our proposal is based on exploiting the available huge photonic nonlinearities that can be generated in specific quantum optical setups. More specifically, we envisage the use of a highly nonlinear waveguide where the necessary nonlinearity will emerge through the strong interaction of the propagating photons to existing emitters in the waveguide. Recent efforts have developed two similar setups in this direction, both capable of implementing our proposal with either current or near future platforms. In these experiments, cold atomic ensembles are brought close to the surface of a tapered fiber [19] or are loaded inside the core of a hollow-core waveguide [20] as shown in Fig. 1(a). The available huge photon nonlinearities due to the (EIT) effect can be used to create situations where the trapped photons obey TM dynamics.

We start by assuming two probe quantum fields of a few photons each, labeled as E_s, $E_{s'}$, are tuned to propagate in the medium. $s, s' = \uparrow, \downarrow$ will denote their different polarizations (later to be representing the spins of the simulated relativistic particles). The effective interaction between photons in these quantum fields will be controlled by external classical fields in a way that realizes a tunable Thirring model. The classical fields consist of two pairs of counterpropagating control lasers with Rabi frequencies $\Omega_{s,\pm}$ with $\pm$ denoting right- and left-moving directions. The atoms placed inside or close to the surface of the waveguide (depends on the implementation) have a ground state $|a\rangle$ and excited states $|b, s\rangle$ and $|d, s, s'\rangle$. Each of the probe pulses is paired with an appropriate classical field and is tuned to an atomic transition such that a typical Λ (EIT) configuration is set up [see Fig. 1(c) and the Supplemental Material [21]].

The process to generate the TM dynamics in our photonic system is as follows. First, the laser-cooled atoms are moved into position so they will interact strongly with incident quantum light fields as done in [19,20]. Initially, resonant to the corresponding transitions, the classical optical pulses with opposite polarizations are sent in from one direction, say the left side. They are injected into the waveguide with the copropagating classical control fields $\Omega_{\uparrow,+}(t)$ and $\Omega_{\downarrow,+}(t)$ initially turned on. As soon as the two quantum pulses completely enter into the waveguide, the classical fields are adiabatically turned off, converting the quantum

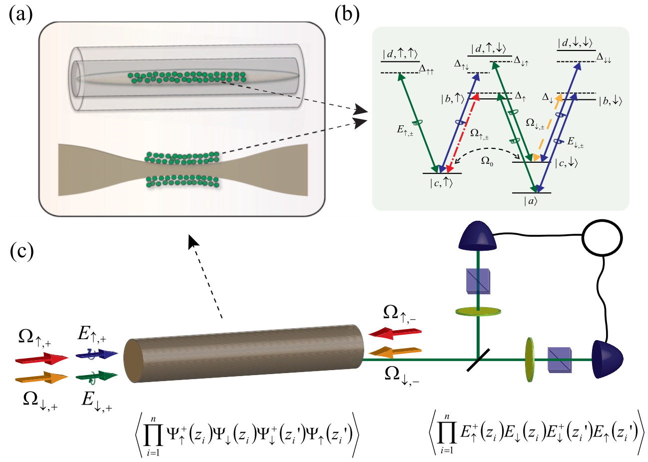

FIG. 1 (color online). An illustration of the possible realization of our proposal. The quantum pulses $\hat{E}_{\uparrow,+}$ and $\hat{E}_{\downarrow,+}$ enter the nonlinear waveguide with different polarizations where they are trapped and made to strongly interact. $\Omega_{\uparrow,\pm}$ and $\Omega_{\downarrow,\pm}$ are two pairs of classical lasers helping to trap and control the pulses. In this regime, the polarized pulses can be made to mimick interacting fermions of opposite spins following the TM dynamics. The scheme is based in engineering the photons' interaction with atoms in the waveguide using EIT techniques. The appropriate relativistic dispersion relation and the relevant TM mass and interaction terms, are fully tunable by tuning optical parameters such as atom-photon detunings and laser strengths. Once the strongly interacting regime is reached, the pulses can be released with all correlations established during the interaction period coherently transferred over to outgoing photon pulses. The latter can be measured using standard quantum optics techniques. At the insets the atomic level structure and a possible implementation using hollow-core fiber or tapered fibers interacting with cold atomic gases.

probes into coherent atomic excitations as done routinely in slow-light experiments. We then adiabatically switch on both counterpropagating classical fields $\Omega_{s,+}$ and $\Omega_{s,-}$ from two sides. The probe pulses become trapped due to the effective Bragg scattering from the stationary classical waves forming the dark state polaritons. This pulse trapping is important in two respects as analyzed in [18,22,23]. First, it significantly extends the interaction time once the optical nonlinearities are turned on, which enables strong quantum correlations to be created from the initially classical probe fields. Second, it allows the group velocity of the probe fields to be tuned to zero. The group velocity dispersion, however, is not zero, which leads to a (tunable) effective mass for the polaritons. At this stage the pulses are noninteracting with the photons expanding freely due to the dispersion. By slowly shifting the d levels we show that the effective masses of the excitations can be kept constant whereas the effective intra- and interspecies repulsions are increased. This drives the system into a strongly interacting regime. This dynamic evolution is possible by keeping for example the corresponding photon detunings Δ_s constant while shifting the d level. Once this correlated state is achieved, the counterpropagating control fields $\Omega_{s,-}$ responsible for trapping the polaritons are slowly turned off. This will release the corresponding quasiparticles by turning them to propagating photons which will then exit the fiber. As all correlations established in the previous step are retained, the propagating wave packets will carry the correlation of the TM dynamics taking place in the medium outside, where they can be probed by appropriately placed photon detectors.

A Thirring model of polarized photons.—In the trapped regime the dark-state polaritons propagating in each direction are related to the electric field operators through the equations $\Psi_{s,+} = \frac{\sqrt{n_z} g}{\Omega_{s,+}} E_{s,+}$ and $\Psi_{s,-} = \frac{\sqrt{n_z} g}{\Omega_{s,-}} E_{s,-}$ [17]. Much like in a high-finesse cavity, however, the Bragg grating created by the counterpropagating control fields couples the dark-state polariton directions together. A more careful analysis shows that the symmetric combination $\Psi_s = \alpha_{s,+}\Psi_{s,+} + \alpha_{s,-}\Psi_{s,-}$ with $\alpha_{s,\pm} = \frac{\Omega_{s,\pm}^2}{\Omega_{s,+}^2 + \Omega_{s,-}^2}$ constitutes the only independent degree of freedom, which obeys the following tunable nonlinear dynamics (see Supplemental Material [21])

$$i\hbar\partial_t \Psi_s = -\frac{\hbar^2}{2m_{\mathrm{nr},s}}\partial_z^2\Psi_s + i\hbar\eta_s\partial_z\Psi_s + \hbar\Omega_0\Psi_{\bar{s}} + \chi_{ss}\Psi_s^\dagger\Psi_s^2 + \chi_{s\bar{s}}\Psi_{\bar{s}}^\dagger\Psi_{\bar{s}}\Psi_s + \text{noise}, \quad (1)$$

where the optically tunable inter- and intraspecies interactions are given by

$$\chi_{ss} = 4\hbar\bar{\Omega}_s^2/(\Delta_{ss} n_z) \quad (2)$$

$$\chi_{s\bar{s}} = 2\hbar\bar{\Omega}_{\bar{s}}^2[2 + \cos(\varphi_{\bar{s}} - \varphi_s)]/(\Delta_{s\bar{s}} n_z). \quad (3)$$

The mass of the particles in the relativistic regime is $m_{0,s} = -\hbar\Omega_0/\eta_s^2$ and $\eta_s = -2v_s\cos 2\varphi_s$ and $\tan^2\varphi_s = \Omega_{s,-}^2/\Omega_{s,+}^2$. We note here that the above mass is different from the "dressed" mass due to the interactions.

The quadratic dispersion term corresponding to the "classical" kinetic energy can be ignored in the regime of interactions we operate (Supplemental Material [21]). In this case, if we define a spinor field $\mathbf{\Psi} = (\Psi_\uparrow, \Psi_\downarrow)^T$, the Hamiltonian generating the last equation becomes the bosonic Thirring model

$$H = \int dz\Big[-i\hbar|\eta|\gamma_1\partial_z + m_0\eta^2)\mathbf{\Psi} + \sum_s \frac{\chi_{ss}}{2}\Psi_s^\dagger\Psi_s^\dagger\Psi_s\Psi_s + \frac{\chi}{2}\bar{\mathbf{\Psi}}\gamma^\mu\mathbf{\Psi}\bar{\mathbf{\Psi}}\gamma_\mu\mathbf{\Psi}\Big], \quad (4)$$

where $\bar{\mathbf{\Psi}} = \mathbf{\Psi}^\dagger\gamma_0$ and the spinor fields satisfy the commutation relations $[\Psi_s(z), \Psi_{s'}(z')] = 0$, $[\Psi_s(z), \Psi_{s'}^\dagger(z')] = \delta(z - z')\delta_{ss'}$. The gamma matrices γ_μ for $\mu = 0, 1$ are chosen as $\gamma_0 = \sigma_x$, $\gamma_1 = i\sigma_y$ with covariant gamma matrices $\gamma^0 = \gamma_0$, $\gamma^1 = -\gamma_1$, and an additional matrix $\gamma_5 = \gamma_0\gamma_1$.

A fermionic TM with tunable mass.—The Hamiltonian in Eq. (4) describes bosonic particles made of stationary light-matter excitations. In order to realize the FTM we will tune to the hard-core regime where the same species interaction is much larger than all other interaction terms. In Fig. 2(a) we calculate and plot the ratio of their strength as a function of the controllable single photon detunings. We see that by appropriate tuning from the corresponding upper atomic states (tuning the laser towards or away from the corresponding states) the system enters a hard-core regime where the same species interaction is much larger than any other term in the Hamiltonian. The particles will then behave as effective fermions in all aspects: identical density-density correlations [24,25] and spectra but different first order correlations due to the sign issue. We note here that as we will discuss later, our optical simulator will be naturally probing intensity-intensity correlations which nicely fits with the above point.

For this regime, we also calculate and plot the individual interaction strengths, denoted β_{ss}^i ($\beta_{s\bar{s}}^i$), as a function of the kinetic energy of the particles for the same values of the tunable optical detunings [Figs. 2(b) and 2(c)]. We see that when the system is optically tuned to the hard-core regime, the intraspecies repulsion is much larger not only the interspecies interaction [Fig. 2(a)] but the kinetic energy as well [Figs. 2(b) and 2(c)]. In plotting [Fig. 2(a)] we have set $m_0 = 0$ with Ω_0 turned off. We note here the simple

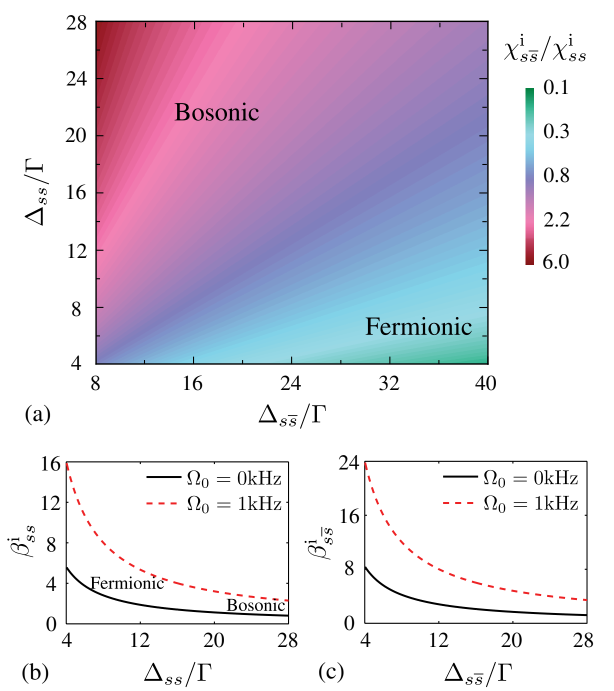

FIG. 2 (color online). (a) The ratio of inter- to intraspecies repulsion as a function of the relevant single photon detunings and the relevant "bosonic" and "fermionic" regimes. (b), (c): The ratio of intra(inter) species interaction to kinetic energy, denoted by β_{ss}^i ($\beta_{s\bar{s}}^i$), as a function of the corresponding single photon detuning.

way of tuning the rest mass of the particles by simply controlling the "connecting" laser.

Probing the TM correlation scalings with photon detectors.—In the following we describe how one could probe the scaling behavior of the correlation functions and from that infer the model's properties and the fascinating renormalization of mass due to the interactions. We use the case of the massless fermionic TM as an example to illustrate our approach. Of course in this case the spectrum of the model and the scattering matrix was explicitly evaluated by the Bethe ansatz [4] and is known. Nevertheless, this regime serves as an excellent check for our quantum simulator. Ultimately, one would like to move to the relatively unexplored regimes of massive or bosonic TM as well as other interesting relativistic theories [26].

In the massless FTM the long-scaled n-point correlation as a function of the distance $z - z'$ and interaction strength $\chi/|\eta|$ was calculated by Coleman in [2] and reads

$$\langle 0|\prod_{i=1}^{n}\Psi_\uparrow^\dagger(z_i)\Psi_\downarrow(z_i)\Psi_\downarrow^\dagger(z_i')\Psi_\uparrow(z_i')|0\rangle = \left(\frac{1}{2}\right)^{2n}\frac{\Lambda^2\prod_{i>j}[(z_i-z_j)^2(z_i'-z_j')^2M^4]^{[1+(\chi/\pi|\eta|)]^{-1}}}{\prod_{i,j}[\Lambda^2(z_i-z_j')^2]^{[1+(\chi/\pi|\eta|)]^{-1}}}. \quad (5)$$

One can extract the two-point correlation with ($n = 1$):

$$\langle 0|\Psi_\uparrow^\dagger(z_1)\Psi_\downarrow(z_1)\Psi_\downarrow^\dagger(z_1')\Psi_\uparrow(z_1')|0\rangle = \frac{\Lambda^2}{4}[\Lambda^2(z_1-z_1')^2]^{-[1+(\chi/\pi|\eta|)]^{-1}}. \quad (6)$$

Λ is the momentum cutoff, which we can estimate for our finite size waveguide of length L containing N interacting particles, to be $\Lambda = \pi n_{\mathrm{ph}}\sin(\chi/|\eta|)/(\chi/|\eta|)$, where $n_{\mathrm{ph}} = N/L$ [4]. In Fig. 3(a) we plot the dependence of the cutoff on the ratio of interactions to kinetic energy for the regime $0 < \chi/|\eta| < \pi$ where modes with unphysically high energy can be excluded. This regime of interactions corresponds to our optical simulator in single photon detuning $\Delta_{ss'}$ ranging up to a few Γ (which is within the required conditions for our slow light setup dynamics [16,17]). In Fig. 3(b), we numerically calculate and plot the two-point correlation as expressed in Eq. (6) as a function of the distance normalized to the inverse of the photonic density for maximum interactions. In these plots, as also in the previous one in Fig. 2, we have assumed the typical values of the optical parameters for the slow light setup, i.e., $|\cos 2\varphi_s| = 0.004$, $n_z = 10^7\ \mathrm{m}^{-1}$, $\Gamma_{1\mathrm{D}} = 0.2\Gamma$, and $\bar{\Omega}_s \simeq 1.5\Gamma$, $0 < \chi/|\eta| < \pi$.

To detect the above correlation in our optical proposal, is enough to observe that it is in fact an effective transverse spin-spin correlation function $\langle S^+(z_1)S^-(z_1')\rangle$ with $S^+(z_1) = \Psi_\uparrow^\dagger(z_1)\Psi_\downarrow(z_1)$ and $S^-(z_1') = \Psi_\downarrow^\dagger(z_1')\Psi_\uparrow(z_1')$. In our case, one would release and map the polaritons to photons by turning off $\Omega_{s,-}$, and then using polarization wave plates to convert Ψ_s into their superpositions $\Psi_{x,\pm}$ and $\Psi_{y,\pm}$ as the pulses exit the medium which then can be measured by probing intensity-intensity correlations (see Supplemental Material [21]). The latter could be done using standard Hanbury-Brown and Twiss type of techniques for different times as time correlations for the propagating photons here map to space of the interacting particles.

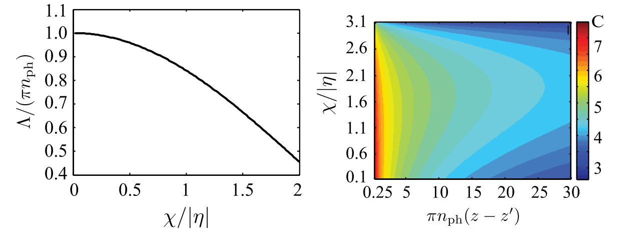

FIG. 3 (color online). The momentum cutoff Λ in unit of πn_{ph} (a) and log-scaled two-point correlations (b) of the massive FTM. As the distances $z - z'$ in (b) increases, the correlations decrease. We simulate the system that each initial quantum pulse contains 10 photons and spreads over after completely entering into 1 cm length fiber.

Discussion.—We have shown how to physically simulate the Thirring model dynamics in a photonic setup where fermions are mimicked by stationary polarized pulses of light in a strongly nonlinear EIT medium. We showed in detail how using coherent light trapping techniques, the photons' dispersion relation can be tuned to the relativistic regime and how to create optically tunable interactions and mass terms. The capability to naturally probe the scaling behavior of the TM correlation functions using standard optical measurements on the emitted pulses, makes our approach complementary to alternative simulations based in cold atom setups. The possibility to first test the device in regimes where analytical solutions are known (massless regime) and then probe regimes with unknown solution (massive FTM and bosonic TM) or other interacting relativistic QFTs makes this extremely interesting in our opinion. Finally, the open nature of our setup and the interaction with the environment through photon loss and driving, should allow for the possibility to investigate QFT theories at finite temperature, an extremely fascinating and well unexplored area of research. We hope that the above characteristics combined with the simplicity of our approach will inspire further experimental and theoretical investigations in using photonic systems for quantum simulations of exotic theories. As one of the future challenges to such photonic simulators, it would be interesting to consider how non-Abelian gauge fields to be implemented on the current system [27–30].

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