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Coherent control of long-distance steady-state entanglement in lossy resonator arrays

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Abstract – We show that coherent control of the steady-state long-distance entanglement between pairs of cavity-atom systems in an array of lossy and driven coupled resonators is possible. The cavities are doped with atoms and are connected through waveguides, other cavities or fibers depending on the implementation. We find that the steady-state entanglement can be coherently controlled through the tuning of the phase difference between the driving fields. It can also be surprisingly high in spite of the pumps being classical fields. For some implementations where the connecting element can be a fiber, long-distance steady-state quantum correlations can be established. Furthermore, the maximal of entanglement for any pair is achieved when their corresponding direct coupling is much smaller than their individual couplings to the third party. This effect is reminiscent of the establishment of coherence between otherwise uncoupled atomic levels using classical coherent fields. We suggest a method to measure this entanglement by analyzing the correlations of the emitted photons from the array and also analyze the above results for a range of values of the system parameters, different network geometries and possible implementation technologies.

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Coupled-cavity arrays have recently been proposed as a new system for realizing schemes for quantum computation [1] and for simulations of quantum many-body systems [2]. More recently driven arrays were considered towards the production of steady-state polaritonic [3] and membrane entanglement [4] under realistic dissipation parameters. Also, an analogy with Josephson oscillations was shown and the many-body properties of the driven array have been recently studied [5].

In this work we examine for the first time the possibility of achieving coherent control of the steady-state entanglement between mixed light-matter excitations (polaritons) generated in macroscopically separated atom-cavity systems. We show explicitly that for a three-pumpedcavity setup, which could be realizable in a variety of cavity QED technologies including photonic crystals, circuit QED, toroidal cavities connected through fibers and coupled defect cavities interacting with quantum dots [6,7], such control is possible (see fig. 1). Light from the connecting waveguides/fibers can directly couple to the photonic modes of the atom-cavity systems through tunneling or evanescent coupling. In each atom-cavity site we assume the interaction and the corresponding nonlinearity to be strong enough to so that either zero or maximally one polariton can be excited¹.

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¹There are several ways to form polaritons through atom-photon interactions. For instance, a two-level atom interacting with cavity photons can exihibit a nonlinear energy spectrum and in the photon-blockade regime only the lowest two levels need to be considered which form the two levels for excitations of polaritons. The ground state of the polariton is $|g, 0\rangle$ and the excited state is the lowest dressed state $(|e, 0\rangle - |g, 1\rangle /)\sqrt{2}$. These two states on resonance are separated by $\omega_{pol} = \omega_0 - g$ with ω_0 the frequency of the uncoupled atomic levels/photon and g the atom-photon coupling strength. There can also be alternatives involving 4-level atoms in each cavity interaction. See also, refs. [8,9]. In the above regime if one calculates

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Fig. 1: (Color online) Schematic representation of a three interacting cavity-atom systems (S_1, S_2, S_3) setup based on a possible implementation using photonic crystals (for illustration purposes only): the connecting waveguides carrying the driving classical fields with phases ϕ_1 , ϕ_2 , ϕ_3 are replaced by fibers or stripline microresonators for different implementations [6,7]. The three waveguides and three driving fields are labeled with the same indices to the phases ϕ_1 , ϕ_2 , ϕ_3 .

The Hamiltonian describing the system written in the rotating frame of the driving lasers is

$$H = \sum_{i=1}^{5} ((\omega_{c,i} - \omega_d) a_i^{\dagger} a_i + (\omega_{p,i} - \omega_d) P_i^{\dagger} P_i) + \sum_{i=1}^{3} J_i (a_i^{\dagger} (P_i + P_{i+1}) + a_i (P_i^{\dagger} + P_{i+1}^{\dagger})) + \sum_{i=1}^{3} (\alpha_i e^{i\phi_i} a_i^{\dagger} + \alpha_i e^{-i\phi_i} a_i),$$
(1)

where the first line is the free Hamiltonian of the waveguides and cavities, with a_i^{\dagger} , a_i the field operators of the single-mode waveguides. P_i^{\dagger} (P_i) are the operators describing the creation (annihilation) of a mixed atomphoton excitation (polariton) at the *i*-th cavity-atom system ($P_4 \triangleq P_1$) (see footnote ¹). The second line describes couplings between cavities and waveguides, with $\omega_{c,i}$, $\omega_{p,i}$ and ω_d the frequencies of the *i*-th waveguide mode, the polariton in *i*-th cavity and the driving fields respectively, and J_i is the coupling strength between the photon mode in the *i*-th waveguide and the adjacent two polaritons. The third line describes the classical driving of the waveguides, where α_i is proportional to the amplitude of the *i*-th driving field with ϕ_i being its phase.

The polaritons and waveguide modes are assumed to decay with rates γ and κ , respectively. The master equation for the polaritonic density matrix, after tracing out the degree of freedom of the waveguide photons ^{1,2} is

$$\dot{\rho} = -i[H_{\text{eff}}, \rho] + \sum_{i=1}^{3} (\Gamma_{i-1} z_{i-1} + \Gamma_i z_i) F_{i,i}^P \rho + \sum_{i=1}^{3} \Gamma_i (F_{i,i+1}^P \rho + F_{i+1,i}^P \rho), \qquad (2)$$

the commutator between the polaritons, a mixed relation will be found which only in the limit of large detunings and/or small couplings leads to a bosonic one. This is the limit where the polaritons are mostly comprised of photons.

²Here, we assume weak pumps: $\alpha_i \leq J_i \ll \kappa$. The method for tracing out the degree of freedom of waveguide modes is similar to the one in ref. [3].

with $H_{\text{eff}} = \sum_{i=1}^{3} (\Gamma_i y_i P_i^{\dagger} P_{i+1} + \Gamma_i x_i (P_i^{\dagger} + P_{i+1}^{\dagger})) + \text{h.c.},$ where h.c. denotes the Hermitian conjugation of its previous summation. $F_{i,j}^P(\rho) = 2P_i\rho P_j^{\dagger} - P_i^{\dagger}P_j\rho - \rho P_i^{\dagger}P_j$, $\Gamma_i = J_i^2\kappa/(\kappa^2 + \Delta_i^2)$, $x_i = \alpha_i e^{i\phi_i}(\Delta_i - i\kappa)/(J_i\kappa)$, $y_i = \Delta_i/\kappa$, $\Delta_{i} = \omega_{c,i} - (\omega_{p,i} + \omega_{p,i+1})/2, \ \omega_{p,4} \triangleq \omega_{p,0}, \ z_{i} = 1 + \gamma/(2\Gamma_{i}),$ $\Gamma_0 \triangleq \Gamma_3$ and $z_0 \triangleq z_3$. It can be seen from eq. (2) that the couplings and detunings between the waveguide and its adjacent two polaritons induce an effective interaction between them given by $\Gamma_i y_i$ (see H_{eff}). The driving on the waveguides is equivalently transferred to the driving on the polaritons ($\Gamma_i x_i$ in H_{eff}), which decay with rates $\Gamma_{i-1}z_{i-1} + \Gamma_i z_i = \Gamma_{i-1} + \Gamma_i + \gamma$. Since Γ_i is related to κ , the polaritons effectively have two channels of decay. They decay directly to the outside with γ and also through the coupling J_{i-1} or J_i $(J_0 \triangleq J_3)$ to the adjacent two leaky waveguides (who also decay by κ). Note that the second channel also mixes the polaritons' operators, as can be seen in the second line of eq. (2). This mixing is actually an essential factor for the entanglement creation among polaritons (the other two essential factors are the interactions among polaritons and the driving on them).

We can now derive the steady state ρ_{ss} by requiring that $\rho_{ss} = 0$ in eq. (2). This is done numerically due to the large number of coupled equations involved (see footnote 1). Next, for a total three-polariton density matrix, we trace out the polaritonic degree of freedom of cavity 1 and calculate the polaritonic entanglement between cavity 2 and 3 using the concurrence as a measure. The concurrence of a two-qubit density matrix ρ is defined [10] as $\max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, where λ_i 's are, in decreasing order, the nonnegative square roots of the moduli of the eigenvalues of $\rho \cdot \tilde{\rho}$ with $\tilde{\rho} = (\sigma_1^y \otimes \sigma_2^y) \cdot \rho^* \cdot (\sigma_1^y \otimes \sigma_2^y)$ and ρ^* is the complex conjugate of ρ . The concurrence $C(\rho_{ss})$ is effectively a function of the parameters x_i , y_i and z_i . We perform a numerical optimization of $C(\rho_{ss})$ by varying these parameters and find that $C(\rho_{ss})$ is larger when $\Gamma_2 \ll \Gamma_1 = \Gamma_3$. For instance, if we assume $\Delta_1 = \Delta_2 =$ $\Delta_3 = 1.5 \times 10^{14} \,\mathrm{Hz}$ and $\kappa = 10^{13} \,\mathrm{Hz}$, the maximum concurrence can reach 0.402 at $x_1 = -x_3 = 1.82, x_2 = 0, y_1 = y_2 =$ $y_3 = 15, z_1 = z_3 = 1.113$, and $z_2 = 114$. These correspond to field amplitudes $\tilde{\alpha}_1 = \tilde{\alpha}_3 = 1.215 \times 10^3$, and couplings $G_1 = -G_3 = 1.0 \times 10^8 \text{ Hz}, \ G_2 = \tilde{\alpha}_2 = 0, \ (\alpha_i = G_i \tilde{\alpha}_i), \ \phi_1 =$ 0, $\phi_3 = \pi$, $\gamma = 10^8$ Hz, $J_1 = J_3 = 1.0 \times 10^{12}$ Hz, $J_2 = 3.16 \times$ 10^{10} Hz. The effective dissipation rates appearing in the initial master equation (eq. (2)) are $\Gamma_1 = \Gamma_3 = 4.42 \times$ 10^8 Hz and $\Gamma_2 = 4.41 \times 10^5$ Hz. These values are consistent with the parameters used in current or near-future technologies [6,7]. Figure 2 shows a plot of the maximum possible concurrence for the polaritonic entanglement of cavity 2 and cavity 3 when the ratio between x_1 and x_3 is varied, with $\Gamma_1 = \Gamma_3$, $\Gamma_2 = 10^{-3}\Gamma_1$, $y_1 = y_3 = 15$, $z_1 = z_3 =$ 1.01 and $z_2 = 11$. Note that since $\Gamma_2 \ll \Gamma_1 = \Gamma_3$, the variation of x_2 and y_2 does not significantly change the value of the concurrence. It can be seen in fig. 2 that $C(\rho_{ss})$, in the case when x_1 and x_3 have opposite signs, is larger than when they have the same signs. $C(\rho_{ss})$ reaches a maximum of 0.417 when $x_3 = -x_1$, *i.e.* the first and third driving



Fig. 2: (Color online) Polar plot $r(\theta)$ of the maximum possible concurrence as the ratio of x_1 and x_3 is varied (fix $y_1 = y_3 =$ 15). $r = C(\rho_{ss})$, $tg\theta = \frac{x_3}{x_1}$ and $sign(x_1) = sign(\cos\theta)$. The insets (1)–(4) are the 3D plots of $C(\rho_{ss})$ as a function of x_1 and y_1 (= y_3) with $\frac{x_3}{x_1}$ fixed to be -1, 1, 0.5, -0.5, respectively.

fields have equal intensity but opposite phases. We also note here that the relation $\Gamma_2 \ll \Gamma_1 = \Gamma_3$ indicates that the coupling between the two cavities in question is much weaker than the coupling between each one and the third cavity. Also the state of the polariton in cavity 1 for the maximum entanglement point is found to be almost a pure state at ground energy level and therefore almost uncorrelated to the polaritons in cavities 2 and 3. Thus, the total density matrix $\rho \approx |\text{ground}| \otimes \rho_{2,3}$. Although this result initially looks counter-intuitive, it can be explained as follows: the maximum entanglement between the two parties, *i.e.* cavities 2 and 3, in a three-party system, is attained when the state of the third party, *i.e.* cavity 1, nearly factorizes in the combined three-party state. The fact that this is happening for strong relative couplings of $J_{12} \equiv J_1$ and $J_{13} \equiv J_3$ compared to $J_{23} \equiv J_2$ is reminiscent of the behavior of a coherent process taking place. One could dare to observe an analogy here with the case of coherently superposing two initially uncoupled ground states in a Λ -type quantum system through an excited state using two classical fields to mediate the interaction [8,9].

The last observation is further justified by observing that $C(\rho_{ss})$ is larger when the first and third driving fields have opposite phases. In fig. 3 we plot $C(\rho_{ss})$ against the phases of driving fields with $z_1 = z_3 = 1.01$ and $z_2 = 11$. When the phase difference is $\phi_1 - \phi_3 = (2k+1)\pi$ (k is an integer), we get again a maximum of 0.417. For general phase relations, an oscillatory behavior characteristic of the expected coherent effect takes place. In simple words,



Fig. 3: (Color online) The concurrence between the polaritons in cavity 2 and cavity 3 as a function of ϕ_1 and ϕ_3 . $x_1 = 1.67e^{i\phi_1}$, $x_3 = 1.67e^{i\phi_3}$. When $\phi_1 - \phi_3 = (2k+1)\pi$ (k is an integer), the concurrence reaches a maximum of 0.417. The upper left figure is the sectional view at $\phi_3 = 0$.



Fig. 4: (Color online) Schematic diagram of the two cavityatom systems in which there are three waveguides carrying the three respective classical laser fields. Note that each waveguide carrying classical fields can also be replaced by fibers or stripline microresonators for implementation technologies [7].

when the two fields are completely out of phase the entanglement is maximized whereas at phase difference $\pi/2$, the two polaritons are completely disentangled. In all other cases, the amount of entanglement lies somewhere in between.

In fig. 4, we study the case of three waveguides coupled to two cavity-atom systems. Here we analyze the polaritonic entanglement between cavity 2 and 3 (relabeled as S_1 and S_2 in fig. 4). The optimization of this entanglement gives similar values of the parameters like the ones used above except that the values for Γ_i are reversed, *i.e.* $\Gamma_2 \gg$ $\Gamma_1 = \Gamma_3$; however, the concurrence can reach a maximum of 0.47. Again the dependence $\phi_1 - \phi_3 = (2k+1)\pi$ (k is an integer) is apparent (see fig. 5). However, if we compare the insets in fig. 3 and fig. 5 for the cross-sectional plots of the concurrence for $\phi_3 = 0$, we see that the plot in fig. 3 has a narrower peak whereas the plot in fig. 5 is broader. This implies that the maximum concurrence for configuration in fig. 4 is substantially more stable against variation in the phases ϕ_1 and ϕ_3 than that in fig. 1.



Fig. 5: (Color online) The concurrence between two cavities (fig. 4) as a function of ϕ_1 and ϕ_3 . $x_2 = y_2 = 0$, $x_1 = 5e^{i\phi_1}$, $x_3 = 5e^{i\phi_3}$, $\Gamma_1 = \Gamma_3 = 1.316 \times 10^8$ and $\Gamma_2 = 10^{10}$. When $\phi_1 - \phi_3 = (2k+1)\pi$ (k is an integer), the concurrence reaches a maximum of 0.470. The upper left figure is the sectional view at $\phi_3 = 0$.

At this juncture, it is worth emphasizing that we now have three different configurations for comparisons: i) two cavities with a single driven waveguide in ref. [3]; ii) two cavities with three driven waveguides as in fig. 4; iii) three cavities with three driven waveguides as in fig. 1. The numerical optimization involving more than three doped-defect cavities does not seem to increase the polaritonic entanglement between any two cavities. Therefore, the above three configurations should be optimal for twoqubit entanglement, corresponding to different values of the dissipation rates parametrized in z. As shown in fig. 6, when z ranges from 1 to 1.221, the maximum concurrence for configuration ii) decreases rapidly from 0.48 to 0.285. This rapid decrease indicates that although configuration ii) can reach higher entanglement than configuration i), yet it is more fragile to the dissipation of the environment (parametrized by γ in z). In comparison, the three-cavity setup is more robust against the increase of dissipation (only when $z \gtrsim 4.03$, its maximum concurrence drops to be the same to that for configuration i)). Therefore, we conclude that cavity 1 in fig. 1 not only coherently mediates between cavities 2 and 3, but it also stabilizes the amount of entanglement between the two cavities.

One could try to employ entanglement witnesses to detect this entanglement [11]. A witness could be constructed from the density matrix corresponding to the maximum value of the concurrence [3] and one could measure the witness along the corresponding spin directions. In coupled-cavity systems to implement the necessary effective spin measurements we can use the usual atomic state measurement techniques employing an external laser tuned to the corresponding polaritonic levels [5] (see footnote ¹). In these measurements the correlations between the polaritons are transferred to



Fig. 6: (Color online) The maximum concurrence vs. z in three configurations: i) two cavities with a single driven waveguide in ref. [3]; ii) two cavities with three driven waveguides as in fig. 4; iii), three cavities with three driven waveguides as in fig. 1 ($z_1 = z_3 = z$, $z_2 = 10^3(z_1 - 1) + 1$). The solid/dashed line is for configuration i)/ii) when 1 < z < 1.221. The dash-dotted line is for configuration iii) when 1 < z < 4.034. The double-dot-dashed line is for configurations i) and ii) when 1.221 < z < 4.034. The dotted line is for all the three configurations when z > 4.034.



Fig. 7: (Color online) (The cross-correlation coefficient $\frac{\langle P_2^{\dagger}P_2 P_3^{\dagger}P_3 \rangle}{\langle P_2^{\dagger}P_2 \rangle \langle P_3^{\dagger}P_3 \rangle}$ for the three-cavity scheme: the minimum value in the cross-correlation coefficient corresponds to maximum concurrence between the cavities.

emitted photons and can thus be detected by analyzing the fluorescent photon spectrum. In the following we plot the cross-correlation coefficient $\frac{\langle P_2^{\dagger}P_2 P_3^{\dagger}P_3 \rangle}{\langle P_2^{\dagger}P_2 \rangle \langle P_3^{\dagger}P_3 \rangle}$ for the three-cavity scheme in fig. 1 as a function of the phase difference between the driving fields 1 and 3 (fig. 7). The plot is consistent with the concurrence plot in fig. 3. What we observe is that when the polaritons are highly entangled the emitted photons come in bunches from each polariton emitter (we note here that the polaritons are continuously pumped).

In this work, we have shown that the long-distance steady-state entanglement in a lossy network of driven light-matter systems can be coherently controlled through the tuning of the phase difference between the driving fields. This entanglement could be measured by analyzing the spectrum of the photons emitted from the cavities. We also found that there exist two optimal setups for generating the maximum available entanglement between two coupled-cavity systems depending on the level of dissipation in the system. Finally, surprisingly enough, in a closed network of three-cavity-atom systems the maximum of entanglement for any pair is achieved even when their corresponding direct coupling is much smaller than their couplings to the third party. This effect is reminiscent of coherent effects found in quantum optics that coherent population transfers between otherwise uncoupled levels through a third level using two classical coherent fields.

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REFERENCES

- [1] ANGELAKIS D. G. et al., Phys. Lett. A, 362 (2007) 377.
- [2] ANGELAKIS D. G., SANTOS M. F. and BOSE S., Phys. Rev. A, 76 (2007) R05709; GREENTREE A. et al., Nat.

Phys., 2 (2006) 856; ROSSINI D. and FAZIO R., Phys. Rev. Lett., 99 (2007) 186401; HUO M. X., LI Y., SONG Z. and SUN C. P., Phys. Rev. A, 77 (2008) 022103; NEIL NA Y. C. et al., Phys. Rev. A, 77 (2008) 031803(R); PATERNOSTRO M., AGARWAL G. S. and KIM M. S., arXiv:0707.0846; IRISH E. K., OGDEN C. D. and KIM M. S., Phys. Rev. A, 77 (2008) 033801.

- [3] ANGELAKIS D. G., BOSE S. and MANCINI S., EPL., 85 (2009) 20007.
- [4] PLENIO M. B. and HUELGA S. F., *Phys. Rev. Lett.*, 88 (2002) 197901; HARTMANN M. J. and PLENIO M. B., *Phys. Rev. Lett.*, 101 (2008) 200503.
- [5] GERACE DARIO et al., Nat. Phys., 5 (2009) 281; TOMADIN
 A. et al., arXiv:0904.4437; CARUSOTTO I. et al., Phys. Rev. Lett., 103 (2009) 033601.
- [6] REITHMAIER J. P. et al., Nature, 432 (2004) 197; ALTUG
 H. and VUCKOVIC J., Appl. Phys. Lett., 84 (2004) 161;
 YOSHIE T. et al., Nature, 432 (2004) 200; HENNESSY K. et al., Nature, 445 (2007) 896; PETER E. et al., Phys. Rev. Lett., 95 (2005) 067401; PRESS DAVID et al., Phys. Rev. Lett., 98 (2007) 117402.
- [7] TAKAO AOKI et al., Nature, 443 (2006) 671; TRUPKE M. et al., Phys. Rev. Lett., 99 (2007) 063601; MAJER J. et al., Nature, 449 (2007) 443.
- [8] SCULLY M. O. and ZUBAIRY M. S., Quantum Optics (Cambridge University Press) 1997.
- [9] BOLLER K.-J., IMAMOGLU A. and HARRIS S. E., Phys. Rev. Lett., 66 (1991) 2593.
- [10] WOOTTERS W. K., Phys. Rev. Lett., 80 (1998) 2245.
- [11] WHITE A. G., JAMES D. F. V., MUNRO W. J. and KWIAT P. G., Phys. Rev. A, 65 (2001) 012301.