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Steady-state entanglement between hybrid light-matter qubits

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Abstract – We study the case of two polaritonic qubits localized in two separate cavities coupled by a fiber/additional cavity. We show that classical driving of the intermediate cavity/fiber can lead to the creation of entanglement between the two ends in the steady state. The stationary nature of this entanglement and its survival under dissipation opens possibilities for its production under realistic laboratory conditions. To facilitate the verification of the entanglement in an experiment we also construct the relevant entanglement witness measurable by accessing only a few local variables of each polaritonic qubit.

Introduction. – Recently, there has been a growing interest in exploiting a certain class of coupled hybrid light-matter systems, namely coupled cavity polaritonic systems, for various purposes such as for realizing schemes for quantum computation [1,2], for communication [3] and for simulations of quantum many-body systems [4–11]. These cavity-atom polaritonic excitations are different from propagating polaritonic excitations in atomic gases and exciton-photon polaritons in solid-state systems [12]. This area is also distinct from those using hybrid light-matter systems in quantum computing where only the matter system (such as an atom or an electron) acts as the qubit. In the latter case the qubits are atoms and light is used exclusively as a connection bus between them [13–19]. In this context there have been promising schemes to produce steady-state entanglement between atoms in distinct cavities [19]. In these proposals, ground states of atoms have been used as the two states of a qubit in order to circumvent decoherence due to spontaneous emission. Additionally, auxiliary atomic levels, external driving fields and an unidirectional coupling between the cavities are required. In polaritonic coupled cavity systems on the other hand, the localized mixed light-matter excitations, or polaritons, allow for the identification of qubits that possess the easy manipulability and measurability of atomic qubits, while also being able to naturally interact whereas separated by distances over which photons can be exchanged between them. Motivated by the rapid experimental progress in Cavity Quantum Electrodynamics and the ability to couple distinct cavities in a variety of systems [20–24], the realization of a system that could produce verifiable, steady-state entanglement between two polaritonic qubits in realistic laboratory conditions would be extremely interesting. Here the challenge is the fact that decoherence emerges from both photonic losses and atomic spontaneous emission due to the mixed nature of the polaritons. Typically, the polaritons would decay to their ground states. Therefore, *a priori*, one may not expect a completely stationary entanglement of two polaritons unless the unavoidable loss of coherence due to both photonic and atomic decays can somehow be "re-injected" into the system.

Here we show that even under dissipation in both the atomic and photonic parts, it is still possible to deterministically entangle two such polaritonic qubits. More precisely, we study the case of two polaritonic qubits coupled by a fiber/additional cavity and show that a *classical driving* can lead to the creation of entanglement between them in the steady state. The stationary nature of this entanglement should make its experimental verification easier. To this end, we also provide a relevant operator (an "entanglement witness" [25]) measurable by only measuring local variables of each polariton.

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The model. – The Hamiltonian describing an array of N identical atom-cavity systems is the sum of the free light and dopant parts and the internal photon and dopant couplings

$$H^{free} = \omega_d \sum_{k=1}^{N} a_k^\dagger a_k + \omega_0 \sum_{k=1}^{N} |e\rangle_k \langle e|, \qquad (1)$$

$$H^{int} = g \sum_{k=1}^{N} (a_k^\dagger \, |g\rangle_k \langle e| + a_k |e\rangle_k \langle g|). \qquad (2)$$

Here $a_k, a_k^\dagger$ are the photonic field operators localized in the k-th system and $|e\rangle_k, |g\rangle_k$ are the excited and ground state of the dopant in the k-th system. Moreover, g is the light-atom coupling strength and ω_d (ω_0) the photonic (atomic) frequencies ($\hbar = 1$ throughout the paper). The $H^{free} + H^{int}$ Hamiltonian can be diagonalized in a basis of mixed photonic and atomic excitations, called *polaritons*. On resonance between atom and cavity, the polaritons are created by operators $P_k^{(\pm,n)\dagger} = |n\pm\rangle_k \langle g, 0\,|$. The states $|n\pm\rangle_k = (|g,n\rangle_k \pm |e, n-1\rangle_k)/\sqrt{2}$ are the polaritonic states (also known as dressed states) with energies $E_n^\pm = n\omega_d \pm g\sqrt{n}$ and $|n\rangle_k$ denotes the n-photon Fock state of the k-th cavity.

It has been shown that in an array of these atom-cavity systems the addition of a hopping photon term $\propto \sum_j (a_j^\dagger a_{j+1} + a_j a_{j+1}^\dagger)$ leads to a polaritonic Mott phase where a maximum of one excitation per site is allowed [5]. This originates from the repulsion due to the photon blockade effect [21]. In this Mott phase, the system's Hamiltonian in the interaction picture results

$$H_I = J \sum_k \left(P_k^{(-,1)\dagger} P_{k+1}^{(-,1)} + P_k^{(-,1)} P_{k+1}^{(-,1)\dagger} \right), \qquad (3)$$

where J is the coupling due to photon hopping from cavity to cavity. Since double or more occupancy of the sites is prohibited, one can identify $P_k^{(-,1)\dagger}$ with $\sigma_k^\dagger = \sigma_k^x + i\sigma_k^y$, where σ_k^x, σ_k^y and σ_k^z stand for the usual Pauli operators. The system's Hamiltonian then becomes the standard XY model of interacting spin qubits with spin up/down corresponding to the presence/absence of a polariton [5].

Let us now consider a linear chain of three coupled cavities with the two extremal ones doped with a two level system as shown in fig. 1(a). Alternatively, as the central cavity in any case is undoped, one can replace it with an optical fiber of short length (so that the distance is greatly increased but the fiber still supports a single mode of frequency near those of the two cavities), which simplifies the setting even further, as shown in fig. 1(b). For the purposes of description, we will use the three-cavity setting remembering that everything applies to the case of two cavities linked by a fiber. The fact that a classical field can drive (*i.e.*, pump energy into) the central cavity in a three-cavity setting (as also shown in fig. 1(a)) is replaced in the fiber setting by a coupler feeding light into the cavity (as also shown in fig. 1(b)).

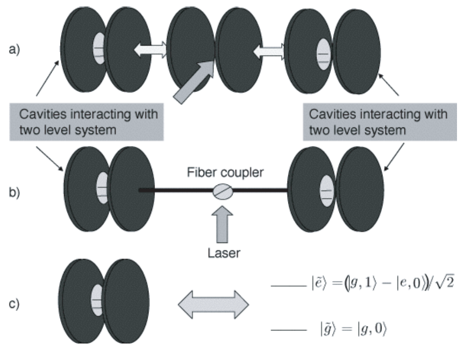

Fig. 1: The system under consideration. a) The cavities are coupled through direct photon hopping. b) The cavities are coupled through a fiber. The extremal cavities in each configuration are interacting with a two-level system that could be an atom or a quantum dot depending the implementation technology used. c) The photon blockade allows for the ground and first dressed states of each atom-cavity system to be treated as a (polaritonic) qubit.

Here it may be worthwhile to emphasize that the above setup and our requirements (*e.g.*, pumping the central cavity) are within current technology. Coupled cavities is a generic system which has, by now, been realized in a variety of experiments ranging from toroidal cavities connected by fibers [22] to fiber coupled cavities on chips [24] to microwave stripline cavities which can couple to each other [26]. In a recent experiment of Bose-Einstein Condensates on chips, cavities where realized by ends of fibers and other cavities to which they can be coupled in the near future where already present on the same chip [27]. In the case when we have a cavity in the middle, light can be fed into the central cavity using different techniques for the various technologies.

For example, for the toroidal case, fibers coming in from any direction tangential to the toroid can couple to the evanescent mode circulating the cavity. So an extra fiber can come in from a direction distinct from the fibers which are coupling this cavity to the other cavities and still couple to the relevant cavity mode. For Fabry-Perot cavities, if an input fiber comes in a sufficiently inclined direction (nearly parallel to the axis connecting the mirrors), then some of the light will eventually couple into the cavity mode. One can also have a highly off resonant scatter such as single or multiple atoms in the central cavity which couple light coming from any direction into the cavity mode, but do not take a relevant role in the Hamiltonian (apart from pumping the central cavity with classical light). For microwave stripline resonators, cases of fig. 1(a) and (b) are indistinguishable because the geometry of the cavities themselves are elongated. Light can be fed in here through a transmission line coming in from a direction perpendicular to the central resonator. When a connecting optical fiber replaces the central cavity (fig. 1(b)) and one needs to feed it in with coherent radiation, one can simply use another fiber to feed in the radiation which is "tunnel coupled" with the connecting fiber [28]. Indeed there are several technologies in existence which can couple one fiber to another so that the field in one can be fed into the other, such as optically coupled optical fibers [29]. This input channel, will, of course, also provide a decay channel for the connecting fiber and in this sense it is equivalent to a central cavity with a certain decay rate.

Let $\sigma_j^\dagger = |1-\rangle_j \langle g,0|$ be the polaritonic spin operators for the end cavities (the index $j=1,2$ labels the two end cavities) and a, $a^\dagger$ the field operators of the empty central cavity. Since the latter is not doped, there the field operators play the role of polariton operators and they couple to polariton operators of the end cavities. Moreover, assuming that the central cavity (or fiber) is driven, the Hamiltonian describing the system dynamics will be

$$H = H_a + H_p + J\left(Sa^\dagger + S^\dagger a\right), \tag{4}$$

where

$$H_a = (\Delta - \delta)\, a^\dagger a, \tag{5}$$

$$H_p = -\delta\left(\sigma_1^z + \sigma_2^z\right), \tag{6}$$

$$S = \left(\sigma_1 + \sigma_2 + \frac{\alpha}{J}\right). \tag{7}$$

Here $\Delta = \omega_{cav} - \omega_{pol}$ is the detuning between the central-cavity mode of frequency ω_{cav} and the polaritonic frequency $\omega_{pol} = \omega_0 - g$. Furthermore, $\delta = \omega_{dri} - \omega_{pol}$ is the detuning between the driving field (of the central cavity) of frequency ω_{driv} and the polaritonic frequency $\omega_{pol} = \omega_0 - g$. Finally, α (hereafter assumed real for the sake of simplicity) is the product of the coupling of the driving field to the central-cavity field (say G) and the amplitude of the driving radiation field (say $\tilde{\alpha}$). We also assume that Δ is much smaller than the atom-light coupling in each of the outer cavities, so that only the ground level $|\tilde{g}\rangle = |g,0\rangle$ and first excited level $|\tilde{e}\rangle = (|g,1\rangle - |e,0\rangle)/\sqrt{2}$ of the polaritons are involved (*i.e.*, the polaritons are still good as qubits).

Suppose that the polaritons decay with the same rate γ (this is the effective decay rate of the polariton due to both the decay of the cavity field and the atomic excited state), and the cavity radiation mode with rate κ. Then, the master equation describing the dynamics of the whole system density operator R will be

$$\dot{R} = \mathbf{L}_a(R) + \mathbf{L}_p(R) + \mathbf{L}_J(R) \tag{8}$$

with the Liouville superoperators

$$\mathbf{L}_a(R) = -i\left[H_a, R\right] + \mathbf{L}_a'(R), \tag{9}$$

$$\mathbf{L}_p(R) = -i\left[H_p, R\right] + \mathbf{L}_p'(R), \tag{10}$$

$$\mathbf{L}_J(R) = -iJ\left[\left(S^\dagger a + Sa^\dagger\right), R\right]. \tag{11}$$

Here $\mathbf{L}_a'$ and $\mathbf{L}_p'$ describe the damping of the radiation mode and the polaritons respectively

$$\mathbf{L}_a'(R) = \kappa\left(2aRa^\dagger - a^\dagger aR - Ra^\dagger a\right), \tag{12}$$

$$\begin{aligned}\mathbf{L}_p'(R) &= \gamma\left(2\sigma_1 R\sigma_1^\dagger - \sigma_1^\dagger\sigma_1 R - R\sigma_1^\dagger\sigma_1\right)\\ &\quad+\gamma\left(2\sigma_2 R\sigma_2^\dagger - \sigma_2^\dagger\sigma_2 R - R\sigma_2^\dagger\sigma_2\right).\end{aligned} \tag{13}$$

We are going to consider the interaction term $J(Sa^\dagger + S^\dagger a)$ as a perturbation and expand the master equation (8) in terms of it. To do so, we cannot consider α increasing at will, but rather $\alpha/J \leqslant 1$. Then, in the decoupled limit $J \to 0$ the cavity field evolves independently of the polaritons. On a timescale κ^{-1} the system relaxes into the state $R(t) \approx r_{ss} \otimes \rho(t)$, with r_{ss} the cavity equilibrium density operator defined by $\mathbf{L}_a(r_{ss}) = 0$, and the polariton operator $\rho(t)$ evolving under the action of $\mathbf{L}_p$. For a finite coupling $J \ll \kappa$ deviations from the factorized form of $R(t)$ are small, but the coupling term $\mathbf{L}_J$ modifies the dynamics of $\rho(t)$. To proceed we adopt a projection operator technique (see, *e.g.*, [30]) and define the projector $\mathbf{P}R = r_{ss} \otimes \mathrm{Tr}_a\{R\}$ and its orthogonal complement $\mathbf{Q} = (\mathbf{1} - \mathbf{P})$. Inserting the decomposition $R(t) = \mathbf{P}R(t) + \mathbf{Q}R(t)$ into eq. (8), we obtain two coupled equations:

$$\mathbf{P}\dot{R}(t) = \mathbf{P}\mathbf{L}_p\mathbf{P}R(t) + \mathbf{P}\mathbf{L}_J\mathbf{Q}R(t), \tag{14}$$

$$\mathbf{Q}\dot{R}(t) = \mathbf{Q}(\mathbf{L}_a + \mathbf{L}_p + \mathbf{L}_J)\mathbf{Q}R(t) + \mathbf{Q}\mathbf{L}_J\mathbf{P}R(t). \tag{15}$$

As the population in the subspace $\mathbf{Q}R$ is damped with a rate κ which is fast compared to the coupling term $\mathbf{Q}\mathbf{L}_J\mathbf{P} \sim J$, we formally integrate eq. (15), insert the result into eq. (14) and expand the final expression up to second order in J. For times $t \gg \kappa^{-1}$ we end up with an effective master equation for the polariton density operator $\rho(t) = \mathrm{Tr}_a\{\mathbf{P}R(t)\}$ which is given by

$$\begin{aligned}&\dot{\rho}(t) = -i[H_p, \rho(t)]\\ &\quad+\int_0^\infty \mathrm{d}\tau\, \mathrm{Tr}_a\{\mathbf{L}_J\mathbf{Q}e^{(\mathbf{L}_a+\mathbf{L}_p)\tau}\mathbf{Q}\mathbf{L}_J e^{-\mathbf{L}_p\tau}\left(r_{ss}\otimes\rho(t)\right)\}.\end{aligned} \tag{16}$$

The second term in eq. (16) describes the effect of the cavity on the polaritonic dynamics. We evaluate this term by inserting the definitions of $\mathbf{L}_a$, $\mathbf{L}_p$ and $\mathbf{L}_J$ given in eqs. (9)–(11) and obtain an effective master equation of the form

$$\dot{\rho} = -i[H_p, \rho] + J\left(T\rho S^\dagger - S^\dagger T\rho + S\rho T^\dagger - \rho T^\dagger S\right). \tag{17}$$

Here we have introduced

$$T = \int_0^\infty \mathrm{d}\tau\, e^{-i(\Delta-\delta)\tau}\, e^{-\kappa\tau}\, S(-\tau), \tag{18}$$

with $S(t) = e^{iH_p t} S e^{-iH_p t}$. In the infinite-bandwidth limit, where κ is large compared to δ, eq. (18) reduces to the simple form $T = S/(\kappa - i\Delta)$. Thus the effective master equation for polaritons becomes

$$\begin{aligned}\dot{\rho} &= -i\left[H_p, \rho\right] + \frac{J^2}{\kappa - i\Delta}\left(S\rho S^\dagger - S^\dagger S\rho\right)\\ &\quad+\frac{J^2}{\kappa + i\Delta}\left(S\rho S^\dagger - \rho S^\dagger S\right).\end{aligned} \tag{19}$$

Rearranging the various terms we get

$$\begin{aligned}\dot\rho &= -i\left[H_{eff},\rho\right]\\ &\quad+(\gamma+\Gamma)\left[2\sigma_1\rho\sigma_1^\dagger-\sigma_1^\dagger\sigma_1\rho-\rho\sigma_1^\dagger\sigma_1\right]\\ &\quad+(\gamma+\Gamma)\left[2\sigma_2\rho\sigma_2^\dagger-\sigma_2^\dagger\sigma_2\rho-\rho\sigma_2^\dagger\sigma_2\right]\\ &\quad+\Gamma\left[2\sigma_1\rho\sigma_2^\dagger-\sigma_1^\dagger\sigma_2\rho-\rho\sigma_1^\dagger\sigma_2\right]\\ &\quad+\Gamma\left[2\sigma_2\rho\sigma_1^\dagger-\sigma_2^\dagger\sigma_1\rho-\rho\sigma_2^\dagger\sigma_1\right],\end{aligned}\tag{20}$$

where $\Gamma=J^2\kappa/(\kappa^2+\Delta^2)$ is the damping rate induced by the radiation mode. Notice that the latter also gives rise to decay channels which mix the polariton operators (last two lines of eq. (20)). Furthermore, the resulting effective Hamiltonian is given by

$$\begin{aligned}H_{eff} &= -\delta\left(\sigma_1^z+\sigma_2^z\right)+\frac{J^2\Delta}{\kappa^2+\Delta^2}\left(\sigma_1^\dagger\sigma_1+\sigma_2^\dagger\sigma_2\right)\\ &\quad+\frac{J^2\Delta}{\kappa^2+\Delta^2}\left(\sigma_1^\dagger\sigma_2+\sigma_2^\dagger\sigma_1\right)\\ &\quad+\frac{J\alpha}{\Delta+i\kappa}\left(\sigma_1^\dagger+\sigma_2^\dagger\right)+\frac{J\alpha}{\Delta-i\kappa}\left(\sigma_1+\sigma_2\right).\end{aligned}\tag{21}$$

The second term is the frequency (Stark) shift and it can be made to cancel with the first one by a proper choice of δ. Hence, we will consider hereafter the effective Hamiltonian as simply given by the second and third lines of eq. (21). It contains an effective interaction term between the two polaritons as well as population driving terms.

Steady-state entanglement. – At the steady state eq. (20) becomes

$$\begin{aligned}0 &= -ix\left[\sigma_1^\dagger+\sigma_2^\dagger,\rho\right]-ix^*\left[\sigma_1+\sigma_2,\rho\right]-iy\left[\sigma_1\sigma_2^\dagger+\sigma_1^\dagger\sigma_2,\rho\right]\\ &\quad+z\left(2\sigma_1\rho\sigma_1^\dagger-\sigma_1^\dagger\sigma_1\rho-\rho\sigma_1^\dagger\sigma_1+2\sigma_2\rho\sigma_2^\dagger-\sigma_2^\dagger\sigma_2\rho-\rho\sigma_2^\dagger\sigma_2\right)\\ &\quad+\left(2\sigma_1\rho\sigma_2^\dagger-\sigma_1^\dagger\sigma_2\rho-\rho\sigma_1^\dagger\sigma_2+2\sigma_2\rho\sigma_1^\dagger-\sigma_2^\dagger\sigma_1\rho-\rho\sigma_2^\dagger\sigma_1\right),\end{aligned}\tag{22}$$

where $x=\alpha(\Delta-i\kappa)/(J\kappa)$, $y=\Delta/\kappa$ and $z=1+\gamma/\Gamma$.

The steady-state solution of eq. (22) can be found by writing the density operator and the other operators in a matrix form, in the basis $\mathbb{B}=\{|\tilde e\rangle_1|\tilde e\rangle_2,|\tilde g\rangle_1|\tilde e\rangle_2,|\tilde e\rangle_1|\tilde g\rangle_2,|\tilde g\rangle_1|\tilde g\rangle_2\}$. Let us parametrize the density operator as

$$\rho_{ss}=\begin{pmatrix}\mathcal{A} & \mathcal{B}_1+i\mathcal{B}_2 & \mathcal{C}_1+i\mathcal{C}_2 & \mathcal{D}_1+i\mathcal{D}_2\\ \mathcal{B}_1-i\mathcal{B}_2 & \mathcal{E} & \mathcal{F}_1+i\mathcal{F}_2 & \mathcal{G}_1+i\mathcal{G}_2\\ \mathcal{C}_1-i\mathcal{C}_2 & \mathcal{F}_1-i\mathcal{F}_2 & \mathcal{H} & \mathcal{I}_1+i\mathcal{I}_2\\ \mathcal{D}_1-i\mathcal{D}_2 & \mathcal{G}_1-i\mathcal{D}_2 & \mathcal{I}_1-i\mathcal{I}_2 & \mathcal{J}\end{pmatrix},\tag{23}$$

where $\mathcal{J}=1-\mathcal{A}-\mathcal{E}-\mathcal{H}$ to respect the requirement $\mathrm{Tr}\{\rho_{ss}\}=1$. The matrix representation of the other operators comes from

$$\sigma_1=\begin{pmatrix}0&0&0&0\\1&0&0&0\\0&0&0&0\\0&0&1&0\end{pmatrix},\quad \sigma_2=\begin{pmatrix}0&0&0&0\\0&0&0&0\\1&0&0&0\\0&1&0&0\end{pmatrix}.\tag{24}$$

By using matrices (23) and (24) in the r.h.s. of eq. (22), we get a single complex matrix M which must be equal to zero. Then, equating to zero the entries of M we get a set of equation for the entries of ρ_{ss}. Since M is Hermitian we can consider

$$M_{jj}=0,\quad j,k=1,2,3,4,\tag{25}$$

$$\Re\{M_{jk}\}=0,\quad k>j,\tag{26}$$

$$\Im\{M_{jk}\}=0,\quad k>j,\tag{27}$$

so to have a set of 16 linear equations. Writing $x=x_1+ix_2$ and solving analytically the set of equations we obtain for $x_2=0$

$$\begin{aligned}\mathcal{A} &= \frac{x_1^4}{d},\\ \mathcal{B}_1 &= 0,\quad \mathcal{B}_2=-\frac{x_1^3z}{d},\\ \mathcal{C}_1 &= 0,\quad \mathcal{C}_2=-\frac{x_1^3z}{d},\\ \mathcal{D}_1 &= -\frac{x_1^2z(1+z)}{d},\quad \mathcal{D}_2=y\frac{x_1^2z}{d},\\ \mathcal{E} &= \frac{x_1^2(x_1^2+z^2)}{d},\\ \mathcal{F}_1 &= \frac{x_1^2z^2}{d},\quad \mathcal{F}_2=0,\\ \mathcal{G}_1 &= -y\frac{x_1z^2}{d},\quad \mathcal{G}_2=-\frac{x_1z(x_1^2+z+z^2)}{d},\\ \mathcal{H} &= \frac{x_1^2(x_1^2+z^2)}{d},\\ \mathcal{I}_1 &= -y\frac{x_1z^2}{d},\quad \mathcal{I}_2=-\frac{x_1z(x_1^2+z+z^2)}{d},\end{aligned}\tag{28}$$

where

$$d=4x_1^4+4x_1^2z^2+z^2\left(y^2+(1+z)^2\right).\tag{29}$$

Notice that for $x_1=0$ we have formally analogous solutions that lead to the same physical result, hence they are not reported.

Now that we know the stationary density matrix, we can use the concurrence as measure of the degree of entanglement [31]:

$$C(\rho_{ss})=\max\{0,\lambda_1-\lambda_2-\lambda_3-\lambda_4\},\tag{30}$$

where λ_i's are, in decreasing order, the nonnegative square roots of the moduli of the eigenvalues of $\rho_{ss}\tilde\rho_{ss}$ with

$$\tilde\rho_{ss}=(\sigma_1^y\sigma_2^y)\,\rho_{ss}^*\,(\sigma_1^y\sigma_2^y),\tag{31}$$

and ρ_{ss}^* denotes the complex conjugate of ρ_{ss}. With respect to the basis $\mathbb{B}$ we have

$$\tilde\rho_{ss}=\begin{pmatrix}\mathcal{J} & -\mathcal{I}_1-i\mathcal{I}_2 & -\mathcal{G}_1-i\mathcal{G}_2 & \mathcal{D}_1+i\mathcal{D}_2\\ -\mathcal{I}_1+i\mathcal{I}_2 & \mathcal{H} & \mathcal{F}_1+i\mathcal{F}_2 & -\mathcal{C}_1-i\mathcal{C}_2\\ -\mathcal{G}_1+i\mathcal{G}_2 & \mathcal{F}_1-i\mathcal{F}_2 & \mathcal{E} & -\mathcal{B}_1-i\mathcal{B}_2\\ \mathcal{D}_1-i\mathcal{D}_2 & -\mathcal{C}_1+i\mathcal{C}_2 & -\mathcal{B}_1+i\mathcal{B}_2 & \mathcal{A}\end{pmatrix},\tag{32}$$

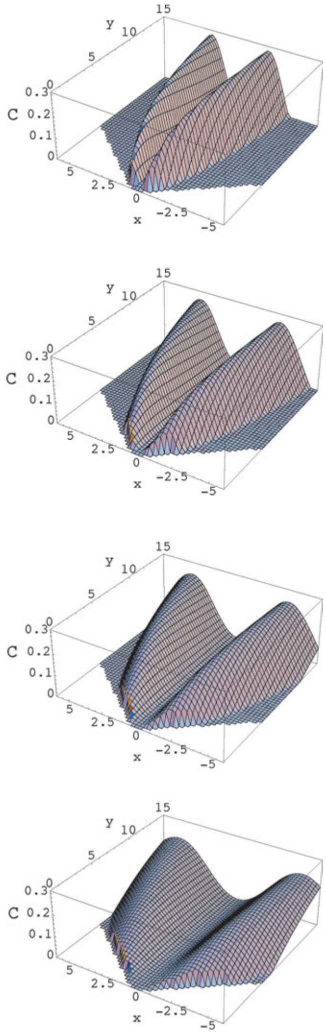

Fig. 2: Concurrence C *vs.* x (real) and y for $z = 1.01$, 2, 3, 5 (from top to bottom). The function is plotted for $|x_1| \leqslant |y|$.

In fig. 2 we show the concurrence as a function of x (real) and y for increasing values of z (top-bottom). Since it is $x_1 = (\alpha/J)y$, we should only consider $|x_1| \leqslant |y|$ due to the assumed condition $\alpha/J \leqslant 1$.

We first notice that by increasing y, the concurrence increases quite slowly, and a maximum amount of entanglement is approximately 0.3 for $y = 15$ and $x_1 = \pm 2.135$. This is similar to the amount of stationary entanglement achievable with an effective interaction of the kind $\sigma_1^z \sigma_2^z$ when combined with an intricate feedback and cascading [17]. Then, by increasing the value of z (*i.e.* the value of γ) there is a broadening effect on the profile of the concurrence. However, the maxima decrease very slowly showing a robustness of entanglement against polaritonic losses.

One could try to employ entanglement witnesses to detect this entanglement [25]. A witness can be constructed from the density matrix corresponding to the maximum value of the concurrence. This would be a traceclass operator W in the Hilbert space of the two polaritonic qubits such that $\mathrm{Tr}[W\rho_{ss}] \geqslant 0$ for all separable states, while $\mathrm{Tr}[W\rho_{ss}] < 0$ for the considered entangled state. Let $\rho_{ss}^{T_1}(C)$ be the partially transposed steady state density operator corresponding to a value $C > 0$ of the concurrence.

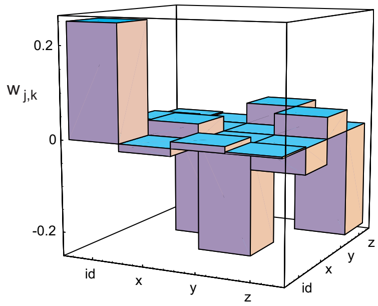

Fig. 3: Elements $w_{j,k}$ of the entanglement witness W detecting the density matrix which maximizes the concurrence in fig. 2, top.

Let also be $|\Psi_-\rangle$ its eigenvector corresponding to the minimal (negative) eigenvalue. Then $W = (|\Psi_-\rangle\langle\Psi_-|)^{T_1}$ Actually, it can be re-expressed in the Pauli decomposition as

$$W = \sum_{j,k=id,x,y,z} w_{j,k}\, \sigma_1^j \otimes \sigma_2^k, \qquad (33)$$

where $\sigma^{id} = I$ and the coefficients turn out to be

$$w_{j,k} = \frac{1}{4}\mathrm{Tr}\left[\sigma_1^j \otimes \sigma_2^k (|\Psi_-\rangle\langle\Psi_-|)^{T_1}\right]. \qquad (34)$$

In fig. 3 we show these coefficients for the entanglement witness coming from the density matrix corresponding to the maximum value of the concurrence in fig. 2. As we can see, the elements with the most significant weights (greater than 0.055) for measuring the witness, correspond to two measurements: $\sigma_1^z \otimes \sigma_2^z$ and $\sigma_1^x \otimes \sigma_2^y$. To implement the σ^z measurement in an experiment, one would need to make a measurement of the population of $|1-\rangle$. This could be done by employing the usual atomic state measurement techniques by the application of a laser field driving $|1-\rangle$ to a third (auxillary) atomic level and observing the fluorescence [32]. For $\sigma^{x,y}$ first a rotation to the x, y basis through the application of a field of frequency $\omega_{pol} = \omega_0 - g$ and then σ^z measurement as above.

The values of x and y used in fig. 2 to get maximal entanglement would correspond to $\Delta = 750J$, $\kappa = 50J$, $G = \gamma = 10^{-4}J$. The pumping coherent field is taken to have roughly $\tilde{\alpha} = 10^4$ photons. J is tunable and depends on the coupling of the photonic modes between neighboring cavities. Assuming this to be of the order of 10^{10} Hz, this would correspond to a cavity dissipation rate $\kappa \approx 10^{13}$ Hz and a polaritonic decay rate $\gamma \approx 10^8$ Hz. These are within the near future in technologies such as coupled toroidal microcavities [22] and coupled superconducting qubits. Coupled defect cavities in photonic crystals arrays are also fast approaching this regime and are extremely suited for the fabrication of regular arrays of many coupled defect cavities interacting with quantum dots [23]. In all technologies, an increase in J, the coupling between the cavity modes, further reduces the requirements on the various lifetimes of the polaritonic and photonic field modes.

Conclusion. – To summarize, this paper presents an example of entangling two qubits in the presence of dissipation despite the fact that each qubit has a continuously decaying state. The entanglement is not transient but stationary, and thereby easy to verify in an experiment, for which there is also a relevant witness. Though the amount of entanglement is not maximal, it is still very interesting as it is for a completely open system. Here although we pump classically, quantum coherence can be established as photons can coherently hop from left to right through the middle cavity. As opposed to the typical case of, say, many-body systems or even the case of two purely atomic qubits in a single cavity or extremely close as to be able to directly interact, here there is the added advantage that the entangled qubits are easily individually accessible (being encoded in distinct atom-cavity systems) for measurements.

Although steady-state entanglement has been pointed out for gas-type systems (*i.e.* systems in which the decoherence processes act locally on the system particles as opposed to strongly coupled systems where they act globally) [33], here is quite intriguing the mechanism from which it generates. It relies on the possibility of compensating losses (through the driving) while maintain the coherence (through the light-matter interaction always on). Hence, a simple classical laser field driving the central cavity/connecting fiber is sufficient to entangle the polaritonic qubits. Loosely speaking, the model presented here can be somehow considered as a complement of that studied in ref. [34], where two driven modes achieve entanglement through an interaction with an atom. Our scheme is apparently more effective to generate steady-state entanglement and it does not show evidences of stochastic resonance effects (at least by increasing γ).

Finally, a scheme feasible with current or near future technology and able to verify polaritonic entanglement as the one we have suggested in this paper, would be a significant first step towards the realization of the plethora schemes to simulate many-body systems and quantum computation using coupled cavities.

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