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Reproducing spin lattice models in strongly coupled atom-cavity systems

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Abstract – In an array of coupled cavities where the cavities are doped with an atomic V-system, and the two excited levels couple to cavity photons of different polarizations, we show how to construct various spin models employed in characterizing phenomena in condensed matter physics, such as the spin-(1/2) Ising, XX, Heisenberg, and XXZ models. The ability to construct networks of arbitrary geometry also allows for the simulation of topological effects. By tuning the number of excitations present, the dimension of the spin to be simulated can be controlled, and mixtures of different spin types produced. The facility of single-site addressing, the use of only the natural hopping photon dynamics without external fields, and the recent experimental advances towards strong coupling, makes the prospect of using these arrays as efficient quantum simulators promising.

Introduction. – The burgeoning field of quantum computation promises much to the science and technology community. While the ability to factor large numbers efficiently may still be some way off, the advances and potential applications brought along with the understanding and control of quantum processes, from beautiful manipulations on minute systems [1] through to coherent many-body operations [2], cannot be underestimated. One of the first such applications is likely to be the simulation of one quantum system with another, more easily manipulated, quantum system. The most general results have been expressed by showing how to simulate one Hamiltonian with another with the help of a series of extremely fast single-qubit rotations, breaking the evolution down into a sequence of stroboscopic pulses which approximate the desired evolution [3], which is known as a Trotter decomposition. However, in physical systems such as optical lattices and ion traps, we possess much more direct ways of simulating a variety of different systems, merely by adjusting periodic potentials using, for example, globally applied lasers [4], making such simulations feasible with current technology.

Of particular interest are models of the form

$$H=\sum_i \vec{B}\cdot\vec{\sigma}_i+\sum_{\langle i,j\rangle}\lambda_z Z_iZ_j+\lambda_x X_iX_j+\lambda_y Y_iY_j, \qquad (1)$$

where $\langle i,j\rangle$ denotes all nearest-neighbour pairs on a lattice of a particular geometry (typically, a 1D chain, or 2D square lattice) and $\vec{\sigma}$ is the vector of Pauli matrices X, Y and Z. There are a number of special cases which are commonly examined. For example, the Ising model ($\lambda_z\neq 0$) in a transverse magnetic field ($B_x\neq 0$) is a simple one-dimensional model which exhibits critical properties. Others include the XX ($\lambda_x=\lambda_y$ and $\lambda_z=0$), Heisenberg ($\lambda_x=\lambda_y=\lambda_z$) and XXZ ($\lambda_x=\lambda_y\neq\lambda_z$). In two-dimensional lattices, such as the hexagonal lattice, simple topological models arise. One possible test-bed for these ideas is an optical lattice setup where the natural Bose-Hubbard Hamiltonian can be manipulated to produce these topological, critical and other effects [5,6]. In addition, they are capable of creating three-body terms and chiral interactions [7].

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Coupled cavities arrays (CCAs) have been initially proposed for the implementation of quantum gates [8]. Recently, intense interest has arisen from the demonstration that a polaritonic Mott transition and a Bose-Hubbard interaction can be generated in these structures [9–11]. In the same work it was shown that the Mott state could be mapped directly to a spin XX model [9]. These papers lead to a plethora of studies on various properties of CCAs in the direction of many body simulations [12], quantum computation [13] and production of photonic entanglement [14]. The study of CCAs provides a theoretical framework that can be implemented using a variety of technologies such as photonic crystals, toroidal microcavities and superconducting qubits [15–17]. Thus, the aforementioned results are not bound to a specific physical system.

In this paper, the aim is to extend this theoretical framework by restricting to the on-resonance, strong coupling, case and examining how one might enrich the simulated model by incorporating more complex atomic structures within the dopants, and by utilising photons of differing polarisations; the goal being to achieve as much of the generalised model described in eq. (1) as possible without resorting to a Trotter decomposition, which imposes additional experimental difficulties. While such decompositions are applicable to the original CCA proposals [9–11], a proposal implementing similar models through the rapid switching of a number of off-resonant time-dependent optical fields followed up by a Trotter expansion has recently been proposed [18]. Coupled cavity arrays are capable of single-qubit addressing, so the corresponding local magnetic fields $\vec{B}$ in a spin model simulation are readily achieved. The key to creating the desired ZZ (which was absent in initial proposals [9–11]) interaction is by suitably selecting the degeneracy of the energy levels of the dopant atoms. We show how an atomic V-system is capable of achieving this. This will provide the additional benefit that simply by tuning the number of excitations in the system, a large range of different, higher-dimensional, spin models can be simulated. The possibility of simulating high-dimensional spins in the presence of strong dissipation using constant external fields is also currently being examined [19]. In the present work, for the case of small dissipation, we present a simpler scheme utilizing just the natural photon hopping dynamics of CCAs, and no time-dependent external fields or detunings.

Atomic V-system. – We start by considering an array of cavities, placed on the vertices of an arbitrary lattice (typically, we consider a regular lattice such as a 1D chain or 2D plane). Each cavity is doped with a single system (which we refer to as an atom), whose energy level structure is that of a ground state, $|g\rangle$ and two degenerate excited states $|A\rangle$ and $|B\rangle$, depicted in fig. 1. Within each lattice site, the Hamiltonian takes the form

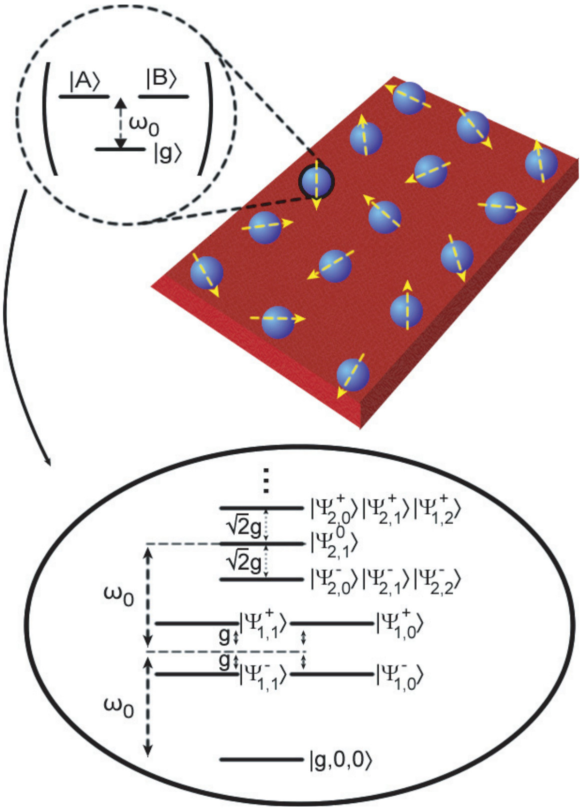

Fig. 1: The atomic V-system on resonance with a cavity. There are two orthogonal photon types, $a^\dagger$ and $b^\dagger$, which only cause transitions to a single level (A or B) from the ground state. This gives rise to a non-linear internal structure on each site.

$$
\begin{aligned}
H_{int} = \omega_0 &\left(aa^\dagger + bb^\dagger + |A\rangle\langle A| + |B\rangle\langle B|\right) \\
&\Delta_A |A\rangle\langle A| + \Delta_B |B\rangle\langle B| \\
&+g\left(|A\rangle\langle g| \otimes a + |g\rangle\langle A| \otimes a^\dagger\right) \\
&+g\left(|B\rangle\langle g| \otimes b + |g\rangle\langle B| \otimes b^\dagger\right),
\end{aligned}
$$

where $a^\dagger$ and $b^\dagger$ create photons of orthogonal polarisations, and are those responsible for promoting the ground state of the atom to the excited states $|A\rangle$ and $|B\rangle$, respectively. Henceforth, we assume that the atomic levels and the cavity are on resonance (*i.e.* the characteristic frequency of the cavity is equal to the frequency of the atomic transitions of the ground state to the excited states; $\Delta_A = \Delta_B = 0$). The strength g represents the strength of the coupling between the cavity and the atom.

In the basis $|\psi, N_A, N_B\rangle$, we can calculate that the (unnormalised) on-site eigenvectors are

$$
\begin{aligned}
|\Psi^0_{S,n}\rangle &= \sqrt{S-n}|A, n-1, S-n\rangle - \sqrt{n}|B, n, S-n-1\rangle \\
|\Psi^\pm_{S,n}\rangle &= \sqrt{n}|A, n-1, S-n\rangle + \sqrt{S-n}|B, n, S-n-1\rangle \\
&\quad \pm\sqrt{S}|g, n, S-n\rangle
\end{aligned}
$$

with energies $S\omega_0$ and $S\omega_0 \pm g\sqrt{S}$ respectively (see fig. 1). N_A and N_B are the number of a and b photons in the cavity, and ψ is the state of the atom. Here, n is an integer index (0 to S) which enumerates the basis within the manifold containing S excitations.

Let us assume that we are working at unit filling fraction, so we expect one excitation per lattice site, meaning that only the states

$$|0\rangle = (|A,0,0\rangle - |g,1,0\rangle)/\sqrt{2},$$
$$|1\rangle = (|B,0,0\rangle - |g,0,1\rangle)/\sqrt{2},$$

are populated. This arises from the observation that there is an energy penalty of $U = (2-\sqrt{2})g$ for moving from one excitation per lattice site to having two excitations on one site, and none on the other.

The individual cavities are coupled together by an interaction

$$H_{hop} = J_a(a_i^\dagger a_{i+1} + a_i a_{i+1}^\dagger) + J_b(b_i^\dagger b_{i+1} + b_i b_{i+1}^\dagger),$$

where J_a, $J_b \ll U$ correspond to the hopping strengths for the two different polarizations of photons between neighbouring cavities [9]. The effect of the coupling can be studied by applying perturbation theory (to the second order) to a pair of neighbouring sites, using the formula

$$H_{eff} = \sum_{a,b\in\{0,1\}^2} |b\rangle\langle a| \sum_{\mu} \frac{\langle b|H_{hop}|\mu\rangle\langle\mu|H_{hop}|a\rangle}{E - E_\mu},$$

where $|\mu\rangle$ are all possible eigenvectors involving 2 excitations on one site, and none on the other. Calculating the relevant matrix elements in the $|0\rangle$, $|1\rangle$ basis we find the effective interaction Hamiltonian

$$H_{eff} = -B_z(\mathbb{1}\otimes Z + Z\otimes\mathbb{1}) - \lambda_z Z\otimes Z - \lambda_x(XX+YY), \tag{2}$$

where

$$\kappa = \frac{31}{32g}\left(J_a^2 + J_b^2\right), \qquad B_z = \frac{5}{8g}\left(J_a^2 - J_b^2\right),$$
$$\lambda_z = \frac{9}{32g}\left(J_a^2 + J_b^2\right), \qquad \lambda_x = \frac{9J_aJ_b}{16g},$$

and we have ignored the term $\kappa\mathbb{1}$ which simply contributes a global phase. The local magnetic fields can be manipulated by applying local Stark fields of our own, thereby leaving an XXZ Hamiltonian where the coefficients λ_z and λ_x are independently tunable (at manufacture of the device). A comparison of the theoretical prediction and an exact diagonalization are depicted in fig. 2. A degree of tunability of the Hamiltonian can be introduced at run-time by varying the detunings of the atomic transitions. However, one must remain in the regime where the detuning is small so that the perturbative expansion still holds, which restricts the range of variation.

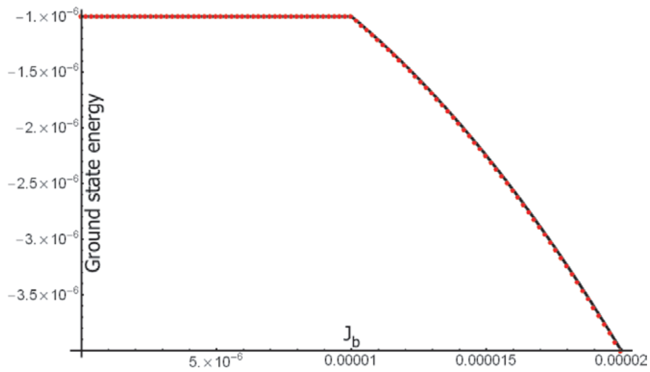

Fig. 2: A comparison of the ground-state energy between a simulation of the full system (dots) and the prediction from perturbation theory (solid line) for 4 cavities, doped with an average of 1 excitation per site. The chosen parameters are $g = 10^{-3}$, $J_a = 10^{-5}$. A phase transition occurs at $J_a = J_b$ between the $|0\rangle^{\otimes 4}$ and $|0\rangle^{\otimes 4}$ ground states. The energies have been scaled to remove the shift of $4\omega_0 - 4g$. To observe other phases, such as the one in the XXZ model requires compensation of the B_z-term by external fields.

Generalised model. – Our hopping terms, with strengths J_a and J_b, effectively describe transmission of photons (between cavities) through a birefringent crystal with the fast and slow axes aligned with the directions a and b. In an optical lattice, one can rotate these axes by applying a Raman transition to the tunnelling potential. In CCAs, the ability to apply this rotation is dependant on the particular realisation under consideration. In a setting where the cavities are connected by optical fibres, such as fibre-coupled micro-toroidal cavities [15], these optical fibres represent the birefringent material that we require, and the optical axes (c and d) can be aligned independently of the directions defined by the atomic transitions (a and b). Moreover, the degree of birefringence (J_a/J_b) and the orientation can potentially be tuned during the experiment by applying an electric field perpendicular to the fibre, and making use of the Kerr effect, rather than having to initialise all of these properties at the point of manufacture. In circuit QED and photonic crystal realisations, however, the hopping comes directly from the overlap of the wavefunctions of the individual sites [16,17], which are thus directly connected to the a/b basis, and it seems unlikely that these will support this generalisation. In cases where this rotation can be achieved, the two sets of axes are unitarily related,

$$\begin{pmatrix} c^\dagger \\ d^\dagger \end{pmatrix} = V \begin{pmatrix} a^\dagger \\ b^\dagger \end{pmatrix},$$

and the simulated Hamiltonian is changed to $(V\otimes V)H_{eff}(V\otimes V)^\dagger$. While this generates a variety of different terms, for example $X_1Y_2 + Y_1X_2$, we are unable to realise the fully anisotropic model XYZ.

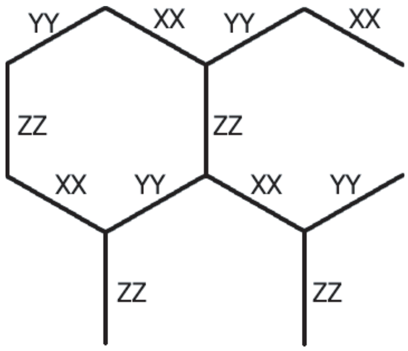

Fig. 3: In a hexagonal lattice, with spin 1/2 located at each vertex, and the indicated couplings, topological effects arise.

One very useful simulation that is introduced due to this rotation is that of the hexagonal lattice [5]. At one limit, this yields the toric code [20], and in another region yields non-Abelian anyons with the aid of an external magnetic field. It is readily formed by setting $J_b = 0$, which implies that $\lambda_x = 0$, and then rotating, along set directions, the remaining term ZZ into XX and YY as required (see fig. 3).

Within the optical lattice community, the possibility of setting $\lambda_x = \lambda_z = 0$ has been explored with a view to eliminating two-body terms, so the leading order of perturbation theory gives three-body interactions. Armed with this toolbox, one could generate many interesting effects such as chiral terms [7]. In optical lattices, this possibility is achieved by using a Feshbach resonance, such that the collisional energies U can be tuned arbitrarily. In the present system, in order to set $\lambda_x = \lambda_z = 0$, one requires $J_a = J_b = 0$, *i.e.* the spins are not coupled, and so three-body terms cannot arise. We might hope to mimic the effect of Feshbach resonances by introducing a detuning between the atom and the cavity, which would serve to shift the energy levels. However, in order to maintain the system's integrity, such a detuning should be $\Delta_{A,B} \ll g$, in which case the shift in energies is unable to entirely cancel the λ_z term.

Higher-spin models. – Unlike the simple two-level dopant considered in [9], changing the average number of excitations per site influences the Hamiltonian that is simulated. If there is an average of S excitations per site, where S is an integer, then there are $S+1$ ground states, $|\Psi^-_{S,n}\rangle$, for $n = 0$ to S, enabling the simulation of a spin-$\frac{1}{2}S$ particle. Again, there is an energy barrier of $U = (2\sqrt{S} - \sqrt{S+1} - \sqrt{S-1})g \sim gS^{-3/2}$ to having any number other than exactly S excitations on each lattice site, so the ground state is the Mott phase for small J/U. All of these models can simulate a Hamiltonian of the form in eq. (2), except with differing coupling coefficients, where the spin operators take on the form of the generalised $SU(2)$ X, Y and Z rotations respectively for the spin $\frac{1}{2}S$. For example, with 2 excitations per site, we realise an array of qutrits interacting through a form described by eq. (2), where X, Y and Z are replaced by the equivalent qutrit operators,

$$J_X = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \Big/ \sqrt{2}, \qquad J_Y = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \Big/ \sqrt{2},$$

$$J_Z = -i[J_X, J_Y]$$

and

$$\kappa = \frac{124\sqrt{2}}{7g}\left(J_a^2 + J_b^2\right), \qquad B_z = \frac{53}{2\sqrt{2}g}\left(J_a^2 - J_b^2\right),$$

$$\lambda_z = \frac{123}{7\sqrt{2}g}\left(J_a^2 + J_b^2\right), \qquad \lambda_x = \frac{123\sqrt{2}J_aJ_b}{7g}.$$

Again, further refinements can be incorporated by implementing the polarization rotations due to the presence of a birefringent material. If the rotation is described by the 2×2 unitary matrix,

$$V = e^{-i\theta(n_xX + n_yY + n_zZ)},$$

then the effective Hamiltonian is rotated by

$$V' = e^{-i\theta(n_xJ_X + n_yJ_Y + n_zJ_Z)}.$$

The functional form of the coupling constants for arbitrary S can be calculated, but is pathological. We note, however, that the leading-order matrix elements are $O(\sqrt{S})$. This significantly adds to the diversity of models that can be efficiently simulated in this simple model, just by changing the number of excitations present in the initial state of the system.

Non-integer filling. – Given that an integer number of excitations, S, per lattice site describes spin-$\frac{1}{2}S$ particles, a non-integer value of average excitations per site potentially describes a blend of different types of particles. Consider the general case where the filling fraction is $S+f$, $0 \leqslant f < 1$. The minimum energy configuration is for a mixture of particles of spin $\frac{1}{2}S$ and $\frac{1}{2}(S+1)$ in the ratio $(1-f) : f$. The analysis of first-order perturbation theory on H_{hop}, yields, for the low-energy dynamics, a swapping of the particles between the sites, governed by the effective Hamiltonian

$$H_{eff}|\Psi^-_{S,i}\rangle|\Psi^-_{S+1,j+1}\rangle =$$
$$\frac{(\sqrt{S}+\sqrt{S+1})^2}{4(S+1)}\Big(J_a\sqrt{(i+1)(j+1)}|\Psi^-_{S+1,i+1}\rangle|\Psi^-_{S,j}\rangle$$
$$+J_b\sqrt{(S-i+1)(S-j)}|\Psi^-_{S+1,i}\rangle|\Psi^-_{S,j+1}\rangle\Big),$$

which should be symmetrised for the possibility where the higher spin particle starts on the left. As already discussed, to first order, there is no interaction between particles of the same type. While we are unaware of a physical phenomenon that this simulates, it completes the analysis of the system in the on-resonance case, and demonstrates the potential that coupled-cavity arrays possess.

Conclusions. – We have described a scheme to realize a family of spin systems in an array of coupled cavities. By introducing a V-configuration to the dopants, the range of nearest-neighbour Hamiltonians that can be simulated is vastly enhanced. With an integer average of S excitations per site, we simulate nearest-neighbour spin-$\frac{1}{2}S$ interactions. For $S=1$, the spin-$\frac{1}{2}$ model allows us to reproduce the Heisenberg, XX and XXZ models as well as those that exhibit both phase transitions and topological features. In the case of a non-integer filling fraction, we simulate a mixture of two particle types interacting. The resultant strong spin-spin coupling and the individual addressability of the separated cavity-atom systems make this approach a promising step towards the realization of quantum simulators for many-body spin problems. Since completing this work, we have become aware of other work which has considered the same V-system [21], which just considered the case of $S=1$, recovering the same results presented here.

The results presented here for the simulation of spin-$\frac{1}{2}S$ particles are exact, up to terms $O(J^3/g^2)$ and in the absence of decoherence ($J=\max(J_a,J_b)$). The primary causes of decoherence are photon loss from the cavities and spontaneous emission from the atoms, whose rates are given by κ and γ, respectively. If decoherence is present in the system, our results remain valid while the corresponding rates are dwarfed by the effective hopping rates, which requires $\sqrt{S}J^2/g \gg \max(\sqrt{S}\kappa,\gamma)$.

Note that even though this scheme is not especially robust against decoherence, such losses cause the system to leave the computational subspace and are thus detectable in the final measurement steps. In order to more successfully combat decoherence, one must utilise a scheme where the states of interest are ground or dark states rather than excited states. Work is progressing in that direction, with results to date requiring the use of a complex scheme employing constant external fields, an elaborate detuning configuration and weakly coupled cavities [19]. Nevertheless, it may be possible to find interesting regimes within our model where quantum phenomena persist, even in the presence of decoherence. For example, in [22], it is described how entanglement can persist in the steady state between a pair of noisy cavities when coupled through a third, pumped, cavity. Although this work makes no reference to how such a scheme might scale, or what information might usefully be extracted, it suggests that further investigation is warranted. The case of non-integer filling fraction is, in fact, more robust to decoherence because it only utilises first-order perturbation theory, and hence we work in a regime where $g \gg JS \gg \max(\sqrt{S}\kappa,\gamma)$. For the case of circuit QED recently $g/\max(\kappa,\gamma)\sim 400$ has been reported [16].

Another intriguing case to study is the atomic V-system in the off-resonant case, and see how the behaviour of the two different photon types mimics those of two-species or single species spinor Bose condensates (see, for example, [23]), which should be different in nature to the non-integer fractional filling discussed here (it has the potential to allow particles to change type).

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