Weaving light-matter qubits into a one way quantum computer

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Abstract. The great advantage of measurement-based quantum computation is that one would simply need the ability to prepare a particular state, known as the cluster state, and subsequently to perform single-qubit measurements on it. Nevertheless, a scalable implementation is yet to be realized. Here, we propose a hybrid light–matter system consisting of coupled cavities interacting with two level systems. Utilizing the stable, individually addressable, qubits resulting from the localized long-lived atom–photon excitations, we demonstrate how to use the natural system dynamics to 'weave' these qubits into a cluster state and propose the implementation of quantum algorithms employing just two rows of qubits. Finally, we briefly discuss the prospects for experimental implementation using atoms, quantum dots or Cooper pair boxes.

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1. Introduction

Measurement-based quantum computation has revolutionized quantum information processing, and the physical systems with which it can be implemented [1]. Quantum computation on cluster states [2] has been proposed in a variety of systems, including linear optics, quantum dots, neutral atoms in optical lattices and flying atom schemes [3]. To date, experiments have been performed using optical lattices [4], where the cluster state can be created but the current lack of individual addressing remains the stumbling block, and linear optics [5, 6], where scalability remains a problem due to the need to generate the initial many-photon state from, for example, high orders of the parametric down conversion process. On the other hand, there have recently been theoretical and experimental breakthroughs into the possibility of direct coupling of high Q cavities and in achieving strong coupling between the cavity mode and an embedded two-level system. At the single cavity level, a plethora of implementations of quantum computing have been proposed and some already been implemented [7]. In the case of coupled cavities, a variety of technologies have been recently employed, namely fibre coupled micro-toroidal cavities interacting with atoms [8], arrays of defects in photonic band gap materials (PBGs) [9] and superconducting qubits coupled through microwave stripline resonators [10]. These have led to proposals for generating control phase gates for photons [11], the production of entanglement [12] and the realization of Mott insulating and superfluid phases [13, 14]. Here, we propose the use of such arrays for the realization of cluster state quantum computation.

2. System description

We start by considering an array of N coupled cavities doped with two-level systems and show how to construct qubits from the hybrid light–matter excitations (polaritons) of each cavity. The atomic states at site k are denoted by $|g\rangle_k$ and $|e\rangle_k$ (we henceforth use the term 'atom' to refer to any relevant two level system which is composed of a ground and an excited state). The Hamiltonian describing the system is the sum of three terms; H^{free} is the Hamiltonian for the free light and dopant parts, H^{int} describes the internal coupling of the photon and dopant within each cavity and H^{hop} encapsulates the effect of light hopping between cavities.

$$H^{\text{free}} = \omega_{\text{d}} \sum_{k=1}^{N} a_k^{\dagger} a_k + \omega_0 \sum_k |e\rangle\langle e|_k \,, \tag{1}$$

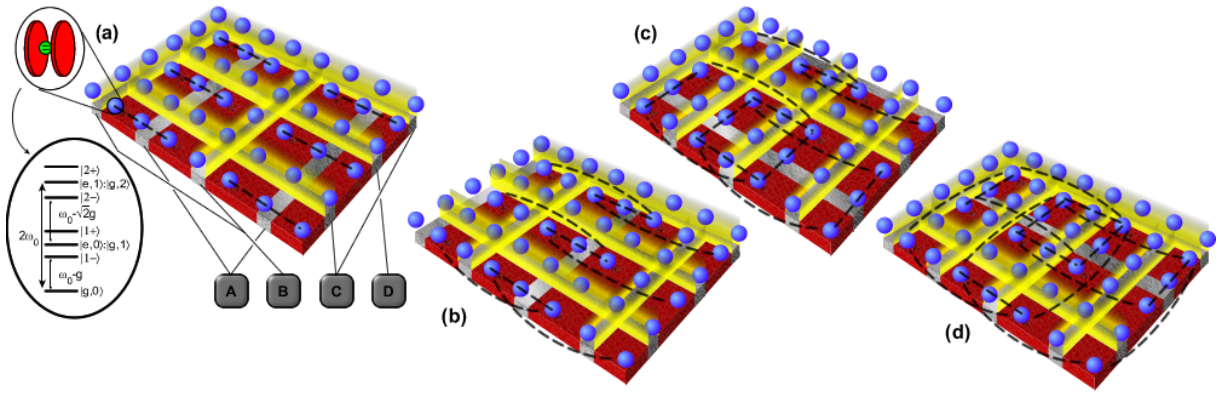

Figure 1. We work with a 2D array of atom–cavity systems. When the atom is on resonance with the cavity, the ground state $|g, 0\rangle$ and the first excited state $|1-\rangle$ of the combined atom–photon (polaritonic) system in each site can be used as qubits. By applying Stark shifts with control electrodes, or properly tuned laser fields, to sets of qubits (the grey gates shown under the qubits), we disable the XY Hamiltonian of a qubit to all of its neighbours. Consecutive application of gates A, B, C and D, depicted in parts (a), (b), (c) and (d), each isolate chains of 3 qubits, realizing controlled-phases and SWAPs (swapping quantum/classical gates) between the qubits at either end of the chain (the dashed lines indicate where the controlled-phase gates have been applied), and are sufficient to connect or weave the three chains, generating a cluster state in parallel across the whole device. Single qubit rotations and measurements are made by properly applying local external fields, utilizing the fact that the cavities can be well separated.

$$H^{\text{int}} = g \sum_{k=1}^{N} (a_k^\dagger \,|\, g\rangle_k \,\langle e\,|_k + a_k \,|\, e\rangle_k \,\langle g\,|_k), \tag{2}$$

$$H^{\text{hop}} = A \sum_{\langle j,k\rangle} (a_j^\dagger a_k + a_j a_k^\dagger), \tag{3}$$

ω_{d} and A are the photon frequencies and hopping rates, respectively and g is the light–atom coupling strength. The term $\langle j, k\rangle$ denotes a sum over nearest-neighbours of the geometry of the array under consideration; we are predominantly interested in the two-dimensional (2D) setting. The $H^{\text{free}} + H^{\text{int}}$ component of the Hamiltonian can be diagonalized in a basis of combined photonic and atomic excitations, called *polaritons* (figure 1). These polaritons, for a system on resonance ($\omega_0 = \omega_{\text{d}}$), are defined by creation operators $P_k^{(\pm,n)\dagger} = |\, n\pm\rangle_k \langle g, 0\,|_k$, where the polaritons of the kth atom–cavity system are given by $|\, n\pm\rangle_k = (|\, g, n\rangle_k \pm |\, e, n-1\rangle_k)/\sqrt{2}$ with energies $E_n^{\pm} = n\omega_{\text{d}} \pm g\sqrt{n}$ (adopting the convention of $\hbar = 1$), and $|\, n\rangle_k$ denotes the n-photon Fock state. As has been shown elsewhere, a polaritonic Mott phase exists in this system where a maximum of one excitation per site is allowed [13]. This originates from the repulsion due to the photon blockade effect [15]. In this Mott phase, the system's Hamiltonian can be written in the interaction picture as $H_{\text{I}} = A \sum_{\langle j,k\rangle} P_j^\dagger P_k + P_j P_k^\dagger$, where $P_k^\dagger = P_k^{(-,1)\dagger}$ (figure 1). As double or more occupancy of the sites is prohibited, one can identify $P_k^\dagger$ with $\sigma_k^+ = \sigma_k^x + \mathrm{i}\sigma_k^y$, where σ_k^x and σ_k^y are the standard Pauli operators. The system's Hamiltonian then becomes the standard

XY model of interacting spin qubits with spin up/down corresponding to the presence/absence of a polariton.

$$H_{\mathrm{I}} = A \sum_{\langle j,k \rangle} \sigma_j^x \sigma_k^x + \sigma_j^y \sigma_k^y . \tag{4}$$

Some applications of XY spin chains in quantum information processing can thus been implemented in this system [16].

3. Cluster state generation

The typical implementation of cluster state quantum computing requires initializing all qubits in a 2D lattice in the $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ state and then performing controlled-phase (CP) gates between nearest-neighbours. In the present system, we have no direct two-qubit gate and the available interaction is not of the Ising type, which straightforwardly gives CP gates [2], but an 'always on' global Hamiltonian coupling of the XY form. Some consideration of similar scenarios has been previously made [17], although these have primarily concentrated on the Heisenberg interaction. In comparison, the technique which we invoke induces entanglement in a more stable way (from the exchange of two effective fermions [18, 19], and hence it is topological in nature), requires fewer control structures but is inapplicable to the case of Heisenberg coupling. Moreover, the strategy that we will outline momentarily is specifically designed to cope with the always-on nature of the interaction—this is an aspect which is often neglected when forming a cluster state either from Hamiltonian interactions such as the Ising model or as the ground state of a Hamiltonian [20]; one must disable the system dynamics once the state has been formed. This requirement can be realized by combining the system's natural dynamics with a protocol where some of the available physical qubits are allocated as gate 'mediators' and the rest as the logical qubits. The mediator atoms can be Stark shifted on- and off-resonance from their cavities through the application of an external field, inhibiting the photon hopping and thereby isolating each logical qubit. The same inhibition of couplings will be used to generate the cluster state. We note here that the error introduced in the step is due to a second-order transition between on-resonance qubits (via a dark-passage through the central off-resonant qubit), which is thus suppressed by a factor of order A/Δ, where $\Delta = \omega_{\mathrm{d}} - \omega_0$ is the detuning of the off-resonant cavity.

Before describing the 4-step global gate sequence to create the cluster state, first observe that to generate the control phase, it is enough to localize chains of three qubits, let them evolve for a time $t_0 = \pi/(2\sqrt{2}A)$ and then apply a measurement on the middle 'mediator' qubit (in the σ^z basis). Depending on the measurement result, $|0\rangle$ or $|1\rangle$, a non-local gate is generated between the remaining two qubits, either SWAP.$(\sigma^z \otimes \sigma^z)$.CP or SWAP.CP, respectively [16, 18]. In both the cases, the gates in addition to the CP are Clifford gates, and can thus be recorded and taken into account during the measurement-based computation with the help of an efficient classical computation. Alternatively, if the mediator starts in a known state, say $|0\rangle$, then measuring it and post-selecting on the $|0\rangle$ outcome acts as a useful form of error suppression against timing errors (perfectly) and some forms of decoherence (giving some improvement). On failure (the $|1\rangle$ result), we can make use of the techniques of Barrett and Kok to fix the error.

Our sequence to generate the cluster state initiates by preparing all qubits in the $|+\rangle$ state through the application of global $\pi/2$ pulse resonant with the $|g, 0\rangle \rightarrow |1-\rangle$ transition. One

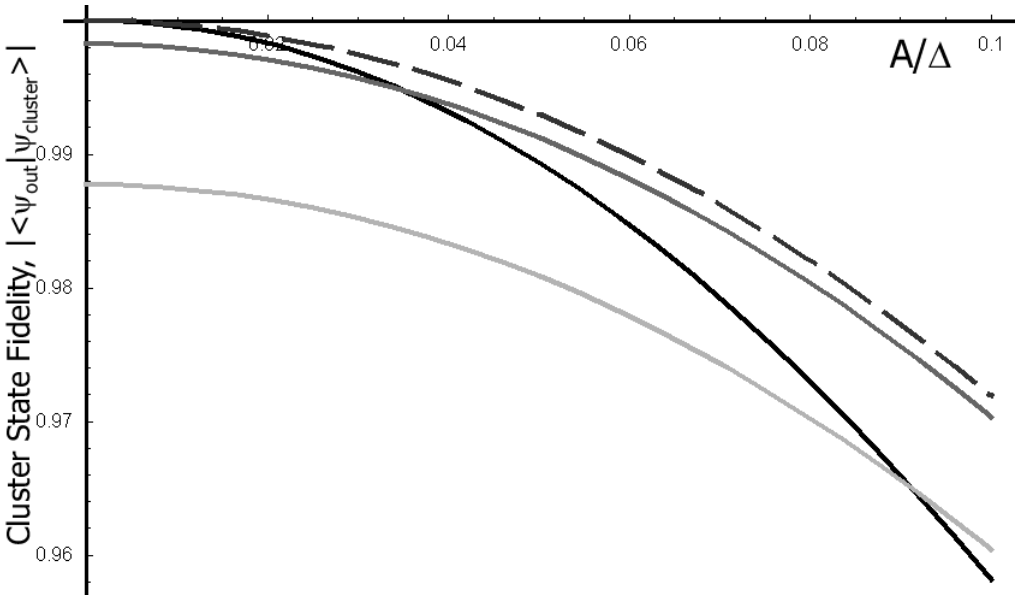

Figure 2. The fidelity of generation of a cluster state on a 3×3 grid of cavities, as the detuning Δ of the mediator off-resonance cavities is varied (in units of the hopping A). The dashed line includes post-selection on getting $|0\rangle$ outcomes when measuring off-resonance qubits. The grey lines also incorporate spontaneous decay and cavity leakage of $0.05A$ (dark) and $0.08A$ (light).

quarter of the sites will be used as logical qubits and the rest as 'mediators' and 'off' qubits interchangeably. All qubits addressed by the gates A–D (figure 1) are, by default, 'off' (meaning that they have been detuned from resonance via a Stark shift), thereby isolating all the qubits. Switching on any one of the four gates thus creates chains of three qubits, which we use to enact a CP between pairs of qubits (separated by a mediator qubit, which was previously off). Consecutive use of each of the gates A–D (figure 1(a) $\rightarrow$ 1(b) $\rightarrow$ 1(c) $\rightarrow$ 1(d)), serves to enact a CP gate between a particular qubit and all of its nearest-neighbours, and this happens in parallel across the whole device. The entire sequence is illustrated in the supplementary video 1, available from stacks.iop.org/NJP/10/023012/mmedia. The measurement sequence is then applied as requested by the cluster state algorithm, utilizing the local accessibility of the sites—in any implementation, the cavity–atom systems are well separated compared to the resolution of the external field used for addressing them [8]–[10].

3.1. Consideration of errors

Apart from the aforementioned effect that comes through second-order perturbation theory, which is the primary assumption we have made in deriving the system dynamics, and which results in an error of order A/Δ, what other practical concerns are likely to limit the usefulness of our scheme? Primarily, our concern should be decoherence, which will typically manifest as cavity leakage and spontaneous emission from the atoms. In figure 2, we calculate the fidelity of generation of a cluster state on a 3×3 grid of cavities, as the detuning Δ of the mediator off-resonance cavities is varied. The dashed line includes post-selection on getting $|0\rangle$ outcomes when measuring off-resonance qubits, while grey lines also incorporate spontaneous decay and cavity leakage. We observe that the fidelity remains larger than 0.97 even when relatively large

values of dissipation are included. More sophisticated schemes have the potential to further reduce the experimental errors. For example, standard Hamiltonian simulation techniques allow us to negate the second-order exchange term due to the off-resonance cavities, simply by repeatedly applying σ_z gates to every second on-resonance triplet throughout the evolution. One might even hope that we could use this coherent effect to enhance the scheme through the use of, for example, optimal control techniques. Most of the errors considered here (cavity leakage, spontaneous emission of the atom and on-off detuning of qubits) are local effects, introducing local noise, which can ultimately be addressed by fault-tolerant techniques [22, 26].

Another class of properties that could be expected to have an effect are timing errors (when the external fields are applied, and how quickly they can be ramped up to maximum strength), and problems with system identification or manufacture. If the system is improperly identified or manufactured, then we will be using an incorrect timescale for the evolution and, as such, it is equivalent to a timing error. Within the difficulties of imperfect system manufacture is the problem of ensuring that the atoms and cavities are on-resonance. However, if they are slightly off-resonance, and we can determine this, external fields can be used to compensate. If this is not possible, then, in fact, it does not cause a problem provided the detuning is sufficiently small that we are still within the Mott insulator phase [13], the only difference will be a slight change in the effective coupling between cavities, and hence another timing effect. Thanks to the mediator spin, specifically our ability to measure it, we have a geometric robustness to timing errors [23], i.e. if our timing error is $O(\delta t)$, the accuracy with which the evolution is achieved is only faulty by $O(\delta t^2)$. Finally, the entangling operation, which is the essential part of the whole scheme, has a topological robustness—tuning the parameters of the Hamiltonian differently leaves the generated phase entirely unaffected provided the evolution has completed successfully. Essentially this is a result of the fact that the presence of $|1\rangle$s in the system can be mapped to the presence of fermions, and it is the topological robustness of the −ve sign appearing when two fermions exchange which we are using [18].

In comparison to other schemes which involve Hamiltonian simulation techniques, i.e. the use of fast single qubit rotations to manipulate one Hamiltonian into appearing like another (specifically the Ising interaction for cluster states), timing errors in our scheme are negligible. This is because Hamiltonian simulation techniques have two important timescales: the small pieces of evolution, $\delta t' \ll 1/A$, and the fast single-qubit rotations which must be short in comparison to $\delta t \ll \delta t'$. The simulation is then accurate to $\delta t'$. In comparison, we only use the fast rotations (in order to implement the detuning), so our evolution is accurate to $O(\delta t^2)$.

4. Implementing algorithms

Initial experimental algorithmic implementations with coupled cavities can be expected to utilize the most basic building block of our scheme, a 3×3 grid of cavities, which allows us to generate a four-qubit cluster state. As with the four-photon cluster state initially used by Walther *et al* [5] this cluster state would be suitable for demonstrating the preparation of an arbitrary one-qubit state, an entangling gate between two qubits, and even the implementation of Grover's search algorithm on two qubits [5]. For example, by applying the local gates $H \otimes H \otimes \sigma_z \otimes \sigma_z$, where H is the Hadamard rotation, we convert the 'box' cluster that the 3×3 grid prepares into the 1D cluster state of four qubits, which is given the interpretation of a single qubit, and measurements on the state yield quantum gates on this single qubit. Moreover, generation of this four qubit cluster state is simpler than generation of an arbitrarily sized cluster state because

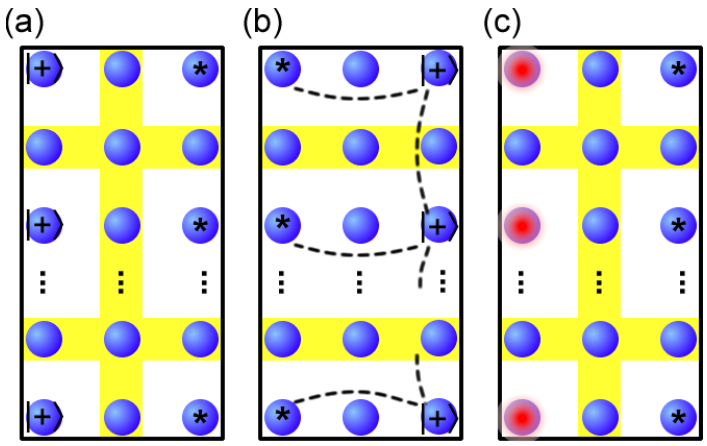

Figure 3. Sequence for minimizing the number of qubits required for a cluster state computation. (a) After the first $n-1$ steps of the algorithm, the first column of qubits is initialized in the $|+\rangle$ state, and the third column, with qubits denoted by $*$, are in the state of output for the first $n-1$ steps of the computation. (b) We use control sequences, bringing mediator qubits on resonance, to convert the $|+\rangle$ states into a cluster state, and to entangle them with the output qubits. The SWAP in the entangling operation moves these output qubits to the first column. (c) Measure the qubits of the first column as corresponds to the nth step of the computation, and reinitialize in the $|+\rangle$ state. The rightmost column yields the output. The sequence then repeats.

we only need two control steps instead of four, thereby keeping us even further within the decoherence time of the system.

Perhaps the next important step would then be to demonstrate Shor's factoring algorithm, the factoring of 15 being the standard demonstration. To implement as a cluster state computation, the six computational qubits [24] translate into the requirement of a cluster state that is eleven qubits wide. Hence, we need an array which is 21 cavities wide. The breadth of the cluster state, which corresponds to time in the circuit model, is a quantity that we can trade against the time taken for the computation. At one extreme, we can create the whole cluster state in one go, with the simple set of four steps already outlined, and we benefit from the large degree of parallelism available to us. This requires a 2D grid of cavities of size 21×311.[5] At the other extreme, a grid of 21×3 cavities suffices. In this case, one starts with the 11×2 cluster state, and performs one time step of measurement (i.e. measure the 11 qubits in one column). The result remains in the other column. We then repeat the cluster state generation process, reinitializing the measured qubits in the cluster state, and performing the next time step (figure 3). This requires 156 consecutive entangling steps, but the reinitializing of the cluster state after measurement eliminates the effect of decoherence over this timescale. Any combination between these two extremes is also possible, and is a necessary property of any scalable implementation of cluster state computation for the sake of preventing decoherence.

[5] To arrive at this required number of gates, we have taken the circuit presented in [24] and converted it into a nearest-neighbour, two-qubit gate algorithm. Hence, the possibility for some small degree of optimization in the number of qubits remains.

Once initial cluster state experiments have been performed, it simply becomes a question of how many cavities one can reasonably couple together. Alternatively, since the two-qubit gate that we can generate is entangling (and hence universal for quantum computation [25]), we can also consider using it directly to implement the circuit model of computation. This has a much smaller overhead of qubits, but instead requires much higher quality cavities. For example, to factor 15 we would only need a 5×3 grid of cavities to give us six computational qubits. However, we would need approximately 15 consecutive entangling steps (we have attempted to minimize this number by allowing as many of the gates to be applied in parallel as possible, and by optimizing the initial labelling of each qubit), hence requiring a time of order $15\pi/(\sqrt{2}A)$. Hence, to reduce the effect of dissipative decay, we require an order of magnitude improvement in the decoherence properties of the qubits to compensate for the increased running time.

5. Experimental implementations

As previously mentioned, there are three primary candidate technologies; fibre coupled microtoroidal cavities [8], arrays of defects in PBGs [9] and superconducting qubits coupled through microwave stripline resonators [10]. In order to achieve the required limit of no more than one excitation per site [13], the ratio between the internal atom–photon coupling and the hopping of photons down the chain should be of the order of $g/A \sim 10^2$–10^1 (A can be tuned while fabricating the array by adjusting the distance between the cavities and g depends on the type of the dopant). In addition, the cavity/atomic frequencies should be $\omega_d, \omega_0 \sim 10^4 g, 10^5 g$ and the losses should also be small, $g/\max(\kappa, \gamma) \sim 10^3$, where κ and γ are cavity and atom/other decay rates. The polaritonic states under consideration are essentially unaffected by decay for a time $10/A$ (10 ns for the toroidal case and 100 ns for microwave stripline resonators). While the decay time of $10/A$ may seem uncomfortably close to the preparation time for a cluster state, $\sqrt{2}\pi/A$, the previously described technique (figure 3) of continuously reforming the cluster state and connecting it to the output of the previous stage allows a continuous computation that exceeds the decay time for an individual cavity. The required parameter values are currently on the verge of being realized in both toroidal microcavity systems with atoms and stripline microwave resonators coupled to superconducting qubits, but further progress is needed. Arrays of defects in PBGs remain one or two orders of magnitude away, but recent developments, and the integrability of these devices with optoelectronics, make this technology very promising as well. In all implementations the cavity systems are well separated by many times the corresponding wavelength of any local field that needs to be applied in the system for the measurement process.

6. Conclusions

In this paper, we have shown how universal quantum computation could be realized in a coupled array of individually addressable atom–cavity systems, where the qubits are given by mixed light–matter excitations in each cavity site. While single-qubit operations can be locally achieved, the only available interaction between qubits is due to the natural system Hamiltonian. We show how to manipulate this to give a CP gate between pairs of qubits. This allows computation either using the circuit model, or a measurement-based computation, the latter being most suited to reducing experimental errors. We have discussed possible architectures for

implementing these ideas using photonic crystals, toroidal microcavities and superconducting qubits and pointed out their feasibility and scalability with current or near-future technology.

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