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The influence of density of modes on dark lines in spontaneous emission

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Abstract

We study two distinct multi-level atomic models in which one transition is coupled to a Markovian reservoir, while another linked transition is coupled to a non-Markovian reservoir. We show that by choosing appropriately the density of modes of the non-Markovian reservoir the spontaneous emission to the Markovian reservoir is greatly altered. The existence of 'dark lines' in the spontaneous emission spectrum in the Markovian reservoir due to the coupling to specific density of modes of the non-Markovian reservoir is also predicted. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

It is now well understood that spontaneous emission of a quantum system depends crucially on the nature of the reservoir with which the system interacts. Spontaneous emission can be modified in a 'tailored' manner by changing the density of modes of the reservoir [1]. An effective method to achieve this is to place atoms in waveguides [2–4], microcavities [5–10] or photonic band gap materials [11–18], where the density of modes differs substantially from that of the free space vacuum. The latter has also attracted attention for its potential for modifying the absorption and dispersion properties of a system [19].

In the above studies the typical scheme involves the interaction of a two-level atom with a reservoir with modified density of modes. Spontaneous emission of this two-level system differs considerably from the free space result and the usual Weisskopf–Wigner exponential decay [20] is violated. In specific cases complete inhibition of spontaneous decay has even been predicted [12,13]. In addition to studies of two-level atoms, there are also investigations of multi-level atoms, where the spontaneous emission from an atomic transition in the modified reservoir influences the spontaneous emission of another atomic transition that interacts with the

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normal free space vacuum [12,16,17]. It is schemes such as these that concerns us in this article. Specifically, we study two distinct multi-level atomic schemes where one transition is considered to decay spontaneously in a modified reservoir and the other decays to a normal free space vacuum. The main interest here is the existence of 'dark lines' (complete quenching of spontaneous emission for specific vacuum modes) in the spontaneous emission spectrum of the *free space* transition due to the coupling of the other transition with the modified reservoir. We note that dark lines in spontaneous emission have been predicted in several laser driven schemes [21–24] and have been experimentally observed [25]. In this article we employ specific models for the modified reservoir density of modes, such as, for example, that which results from an isotropic photonic band gap model with (or without) defects. We show that dark lines appear in the spontaneous emission spectrum as a consequence of the structure of the modified reservoir.

This article is organized as follows: in the next section we consider a three-level, Λ-type atomic system, with one transition spontaneously decaying in the normal free space vacuum and the other transition decaying in a modified reservoir. We study the spontaneous emission spectrum of the free space transition for four different density of modes of the modified reservoir and show that dark lines can occur in the spectrum due to the structure of the modified reservoir. We note that John and Quang [12] have studied the same atomic system as us, using specifically the appropriate density of modes obtained near the edge of an isotropic photonic band gap [see Eq. (12)]. However, in their study they focussed on the phenomenon of dynamical splitting in the spectrum and not on the existence of dark lines, which is the main phenomenon discussed in our article. Zeros and splittings in spectra should not be confused, although they relate to each other in the limit of strong coupling. In Section 3 we consider a laser-driven extension of the previous system and show that dark lines can occur in this system, too. In this case the dark lines originate from either laser-induced or modified vacuum-induced mechanisms. Finally, we summarize our findings in Section 4.

2. First case: Λ-type scheme

We begin with the study of the Λ-type scheme, shown in Fig. 1(a). This system is similar to that used by Lewenstein et al. [3] and by John and Quang [12]. The atom is assumed to be initially in state $|2\rangle$. The transition $|2\rangle \leftrightarrow |1\rangle$ is taken to be near resonant with a modified reservoir (this will be later referred to as the non-Markovian reservoir), while the transition $|2\rangle \leftrightarrow |0\rangle$ is assumed to be occurring in free space (this will be later referred to as the Markovian reservoir). The spectrum of this latter transition is of central interest in this article. The Hamiltonian which describes the dynamics of this system, in the interaction picture and the rotating wave approximation (RWA), is given by (we use units such that $\hbar = 1$),

$$H = \sum_{\lambda} g_{\lambda} e^{-i(\omega_{\lambda} - \omega_{20})t} |2\rangle \langle 0| a_{\lambda} + \sum_{\kappa} g_{\kappa} e^{-i(\omega_{\kappa} - \omega_{21})t} |2\rangle \langle 1| a_{\kappa} + \text{H.c.} \tag{1}$$

Here, g_{κ} denotes the coupling of the atom with the modified vacuum modes (κ) and g_{λ} denotes the coupling of the atom with the free space vacuum modes (λ). Both coupling strengths are taken to be real. The energy separations of the states are denoted by $\omega_{ij} = \omega_i - \omega_j$ and ω_{κ} (ω_{λ}) is the energy of the κ (λ)-th reservoir mode.

The description of the system is given using a probability amplitude approach. We proceed by expanding the wave function of the system, at a specific time t, in terms of the 'bare' state vectors such that

$$|\psi(t)\rangle = b_2(t)|2,\{0\}\rangle + \sum_{\lambda} b_{\lambda}(t)|0,\{\lambda\}\rangle + \sum_{\kappa} b_{\kappa}(t)|1,\{\kappa\}\rangle . \tag{2}$$

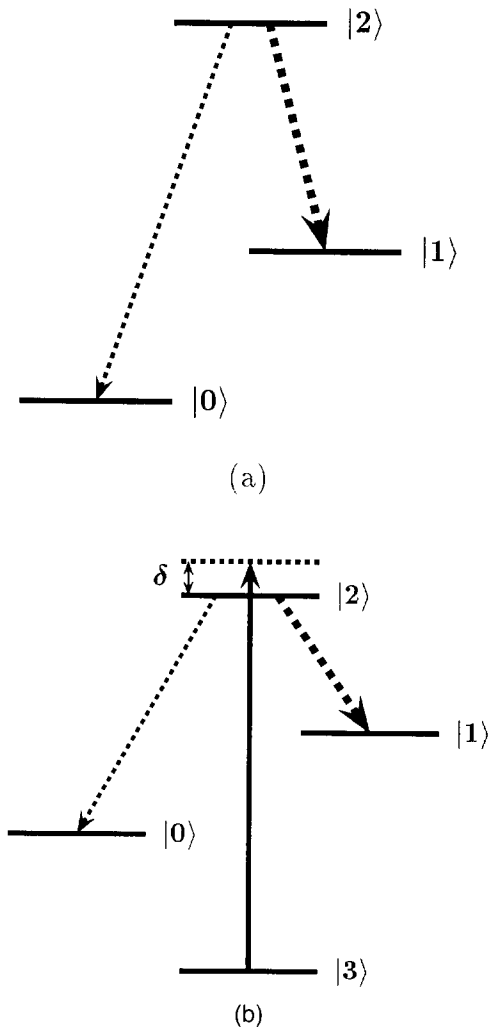

Fig. 1. The two systems under consideration. In (a) the thick dashed line denotes the coupling to the modified reservoir and the thin dashed line denotes the coupling to the Markovian reservoir. The same hold in (b), and in addition the solid line denotes the coupling by the laser field.

Substituting Eqs. (1) and (2) into the time-dependent Schrödinger equation we obtain

$$\mathrm{i}\dot{b}_2(t) = \sum_{\lambda} g_\lambda b_\lambda(t)\mathrm{e}^{-\mathrm{i}(\omega_\lambda-\omega_{20})t} + \sum_{\kappa} g_\kappa b_\kappa(t)\mathrm{e}^{-\mathrm{i}(\omega_\kappa-\omega_{21})t}, \tag{3}$$

$$\mathrm{i}\dot{b}_\lambda(t) = g_\lambda b_2(t)\mathrm{e}^{\mathrm{i}(\omega_\lambda-\omega_{20})t}, \tag{4}$$

$$\mathrm{i}\dot{b}_\kappa(t) = g_\kappa b_2(t)\mathrm{e}^{\mathrm{i}(\omega_\kappa-\omega_{21})t}. \tag{5}$$

We proceed by performing a formal time integration of Eqs. (4) and (5) and substitute the result into Eq. (3) to obtain the integro-differential equation

$$\dot{b}_2(t) = -\int_0^t \mathrm{d}t' b_2(t') \sum_\lambda g_\lambda^2 \mathrm{e}^{-\mathrm{i}(\omega_\lambda - \omega_{20})(t-t')} - \int_0^t \mathrm{d}t' b_2(t') \sum_\kappa g_\kappa^2 \mathrm{e}^{-\mathrm{i}(\omega_\kappa - \omega_{20})(t-t')} . \tag{6}$$

Because the reservoir with modes λ is assumed to be Markovian, we can apply the usual Weisskopf–Wigner result [20] and obtain

$$\sum_\lambda g_\lambda^2 \mathrm{e}^{-\mathrm{i}(\omega_\lambda - \omega_{20})(t-t')} = \frac{\gamma}{2}\delta(t-t') . \tag{7}$$

Note that the principal value term associated with the Lamb shift which should accompany the decay rate has been omitted in Eq. (7). This does not affect our the results, as we can assume that the Lamb shift is incorporated into the definition of our state energies. For the second summation in Eq. (6), the one associated with the modified reservoir modes, the above result is not applicable as the density of modes of this reservoir is assumed to vary much quicker than that of free space. To tackle this problem, we define the following kernel

$$K(t-t') = \sum_\kappa g_\kappa^2 \mathrm{e}^{-\mathrm{i}(\omega_\kappa - \omega_{21})(t-t')} \approx g^{3/2} \int \mathrm{d}\omega \rho(\omega) \mathrm{e}^{-\mathrm{i}(\omega - \omega_{21})(t-t')} , \tag{8}$$

which is calculated using the appropriate density of modes $\rho(\omega)$ of the modified reservoir. In Eq. (8), g denotes the coupling constant of the atom to the non-Markovian reservoir. Using Eqs. (7) and (8) into Eq. (6) we obtain

$$\dot{b}_2(t) = -\frac{\gamma}{2} b_2(t) - \int_0^t \mathrm{d}t' b_2(t') K(t-t') . \tag{9}$$

The long time spontaneous emission spectrum in the Markovian reservoir is given by $S(\delta_\lambda) \propto |b_\lambda(t \to \infty)|^2$, with $\delta_\lambda = \omega_\lambda - \omega_{20}$ [21,24]. We calculate $b_\lambda(t \to \infty)$ with the use of the Laplace transform [26] of the equations of motion. Using Eq. (4) and the final value theorem [26] we obtain the spontaneous emission spectrum as $S(\delta_\lambda) \propto \gamma |\lim_{s \to -\mathrm{i}\delta_\lambda} B_2(s)|^2$, where $B_2(s)$ is the Laplace transform of the atomic amplitudes $b_2(t)$ and s is the Laplace variable. This in turn, with the help of Eq. (9), reduces to

$$S(\delta_\lambda) \propto \frac{\gamma}{\left| -\mathrm{i}\delta_\lambda + \gamma/2 + \tilde{K}(s \to -\mathrm{i}\delta_\lambda) \right|^2} , \tag{10}$$

where $\tilde{K}(s)$ is the Laplace transform of $K(t)$, which yields

$$\tilde{K}(s) = g^{3/2} \int \mathrm{d}\omega \frac{\rho(\omega)}{s + \mathrm{i}(\omega - \omega_{21})} . \tag{11}$$

Therefore, in order to calculate the spontaneous emission spectrum in the Markovian reservoir we need to calculate $\tilde{K}(s)$. This will be done for different models of the density of modes $\rho(\omega)$ of the non-Markovian reservoir.

We begin by considering the non-Markovian reservoir to be that obtained near the edge of an isotropic photonic band gap model [12,13]. Then,

$$\rho(\omega) = \frac{1}{\pi}\frac{1}{\sqrt{\omega - \omega_g}}\Theta(\omega - \omega_g), \tag{12}$$

where ω_g is the gap frequency and Θ is the Heaviside step function. In this case, Eq. (11) leads to

$$\tilde{K}(s) = \frac{g^{3/2}}{\sqrt{\mathrm{i}s - \delta_g}}, \tag{13}$$

with $\delta_g = \omega_g - \omega_{21}$. The spontaneous emission spectrum then reads

$$S(\delta_\lambda) \propto \gamma\left|\frac{\sqrt{\delta_\lambda - \delta_g}}{(-\mathrm{i}\delta_\lambda + \gamma/2)\sqrt{\delta_\lambda - \delta_g} + g^{3/2}}\right|^2. \tag{14}$$

Obviously, the spectrum exhibits a zero (i.e. predicts the existence of a dark line), if $\delta_\lambda = \delta_g$. This is purely an effect of the above density of states, and the non-Markovian character of the reservoir. In the case of a Markovian reservoir the spontaneous emission spectrum would obtain the well-known Lorentzian profile and no dark line would appear in the spectrum. The behaviour of the spectrum is shown in Fig. 2 for different values of the detuning from the threshold. The spectrum has two well-separated peaks and the dark line appears at the predicted value. We note that a spectrum of this form has also been derived by John and Quang [12]; however, in their study they did not mention the existence of the dark line (i.e. the zero in the spectrum and its spectral position), which is of central importance in this article.

The density of the modes of Eq. (12) is a special case of a more general family of density of modes those given by

$$\rho(\omega) = \frac{1}{\pi}\frac{\sqrt{\omega - \omega_g}}{\epsilon + \omega - \omega_g}\Theta(\omega - \omega_g), \tag{15}$$

where ϵ is usually referred to as the smoothing parameter. Such a density of modes has been used in studies of atoms in waveguides [2,3] microcavities [5,8] and photonic band gap materials [13]. Eq. (12) is recovered by taking the limit $\epsilon \to 0$ in Eq. (15). For the above density of modes, Eq. (15) we obtain

$$\tilde{K}(s) = \frac{g^{3/2}}{\mathrm{i}\sqrt{\epsilon} + \sqrt{\mathrm{i}s - \delta_g}}, \tag{16}$$

$$S(\delta_\lambda) \propto \gamma\left|\frac{\mathrm{i}\sqrt{\epsilon} + \sqrt{\delta_\lambda - \delta_g}}{(-\mathrm{i}\delta_\lambda + \gamma/2)\left(\mathrm{i}\sqrt{\epsilon} + \sqrt{\delta_\lambda - \delta_g}\right) + g^{3/2}}\right|^2. \tag{17}$$

So, in this 'smoothed' case, no dark line appears in the spectrum. This is shown in Fig. 3, where the zero disappears from the spectrum. The only case for which spontaneous emission spectrum will give zero is that of $\epsilon = 0$ and $\delta_\lambda = \delta_g$ which, of course, reduces to the previous result of Eq. (14).

The above two density of modes describe 'pure' materials (i.e. materials with no defects). In reality, defects exist in waveguides, microcavities or photonic band gap materials which exhibit gaps in their density of modes. These defects lead to narrow-linewidth, well-localized modes in the gaps of the above structures. Their density

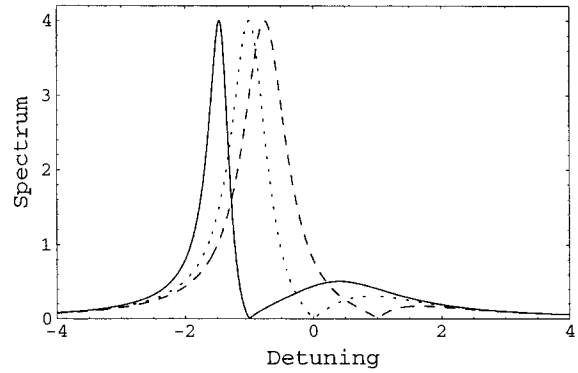

Fig. 2. The spontaneous emission spectrum $S(\delta_\lambda)$ (in arbitrary units) given by Eq. (14) for parameters $g = 1$, and $\delta_g = 0$ (dotted curve); $\delta_g = 1$ (dashed curve); $\delta_g = -1$ (full curve). All parameters are in units of γ.

of modes can be described by either a delta function [11] or a narrow Lorentzian [13]. We will now combine the density of modes given by Eq. (12) and that given by a defect structure. In the case that we choose a delta function density of modes for the defect structure we obtain

$$\tilde{K}(s) = \frac{g^{3/2}}{\sqrt{\mathrm{i}s - \delta_g}} + \frac{g_1^2}{s + \mathrm{i}\delta_c}, \tag{18}$$

$$S(\delta_\lambda) \propto \gamma \left| \frac{\sqrt{\delta_\lambda - \delta_g}}{(-\mathrm{i}\delta_\lambda + \gamma/2)\sqrt{\delta_\lambda - \delta_g} + g^{3/2} + \mathrm{i}\, g_1^2 \dfrac{\sqrt{\delta_\lambda - \delta_g}}{\delta_\lambda - \delta_c}} \right|^2, \tag{19}$$

where $\delta_c = \omega_c - \omega_{21}$ is the defect mode-atom detuning (defect mode at frequency ω_c) and g_1 is the defect mode-atom coupling constant. In this case the spectrum exhibits two dark lines one at $\delta_\lambda = \delta_g$ and another at $\delta_\lambda = \delta_c$. In Fig. 4 we present the spontaneous emission spectrum described by Eq. (19) for the same parameters

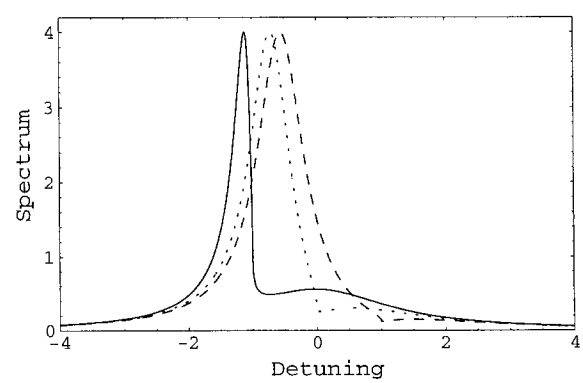

Fig. 3. The spontaneous emission spectrum $S(\delta_\lambda)$ given by Eq. (17) for parameters $g = 1$, $\epsilon = 0.3$, and $\delta_g = 0$ (dotted curve); $\delta_g = 1$ (dashed curve); $\delta_g = -1$ (full curve).

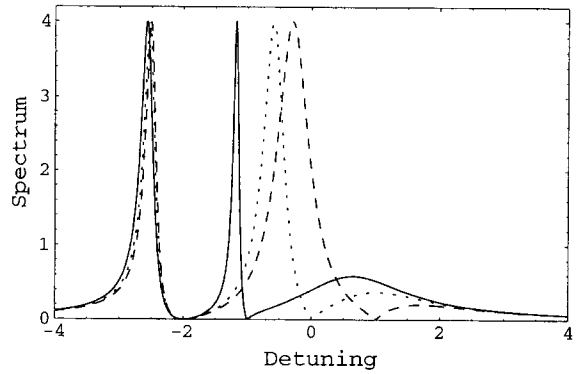

Fig. 4. The spontaneous emission spectrum $S(\delta_\lambda)$ given by Eq. (19) for parameters $g = 1$, $g_1 = 1$, $\delta_c = -2$, and $\delta_g = 0$ (dotted curve); $\delta_g = 1$ (dashed curve); $\delta_g = -1$ (full curve).

that used in Fig. 2. The spectrum obtains now a very pronounced third peak and two dark lines at the predicted values. For a Lorentzian density of modes of the defect structure we get

$$\tilde{K}(s) = \frac{g^{3/2}}{\sqrt{\mathrm{i}s - \delta_g}} + \frac{g_1^2}{s + \mathrm{i}\delta_c + \gamma_c/2}, \tag{20}$$

$$S(\delta_\lambda) \propto \gamma \left| \frac{\sqrt{\delta_\lambda - \delta_g}}{(-\mathrm{i}\delta_\lambda + \gamma/2)\sqrt{\delta_\lambda - \delta_g} + g^{3/2} + g_1^2 \dfrac{\sqrt{\delta_\lambda - \delta_g}}{\mathrm{i}(\delta_c - \delta_\lambda) + \gamma_c/2}} \right|^2, \tag{21}$$

with γ_c being the width of the Lorentzian describing the defect mode. As in the case of the density of modes given by Eq. (12), in this case too, there is a single dark line in the spectrum at $\delta_\lambda = \delta_g$. However, the behaviour of the spectrum is quite different in this case compared to that shown in Fig. 2, as can be seen in Fig. 5.

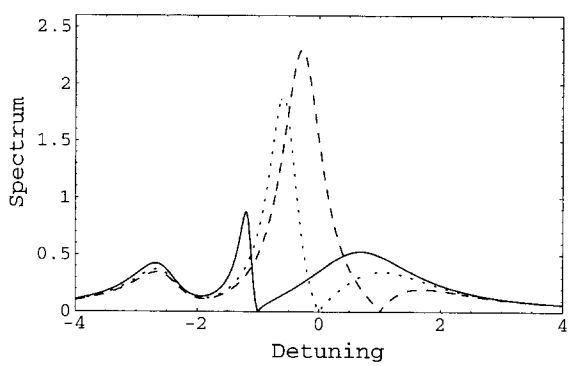

Fig. 5. The spontaneous emission spectrum $S(\delta_\lambda)$ given by Eq. (21) for parameters $g = 1$, $g_1 = 1$, $\gamma_c = 1$, $\delta_c = -2$, and $\delta_g = 0$ (dotted curve); $\delta_g = 1$ (dashed curve); $\delta_g = -1$ (full curve).

3. Second case: laser-driven scheme

In this section we turn to the study of the laser-driven system shown in Fig. 1(b). This system is composed of a ground state $|3\rangle$ which is coupled by a laser field to the excited state $|2\rangle$. State $|2\rangle$ can spontaneously couple to either state $|0\rangle$ via interaction with a Markovian reservoir, or to state $|1\rangle$ via interaction with a non-Markovian reservoir, as in the previous section. The spontaneous emission spectrum in the Markovian reservoir is our main concern in this section, too. The Hamiltonian of this system, written in the interaction picture and under the RWA reads

$$H = \Omega|3\rangle\langle 2|e^{i\delta t} + \sum_{\lambda} g_{\lambda} e^{-i(\omega_{\lambda}-\omega_{20})t}|2\rangle\langle 0|a_{\lambda} + \sum_{\kappa} g_{\kappa} e^{-i(\omega_{\kappa}-\omega_{21})t}|2\rangle\langle 1|a_{\kappa} + \text{H.c.}. \tag{22}$$

Here, Ω is the Rabi frequency (assumed real) and $\delta = \omega - \omega_{23}$ the laser detuning, with ω being the laser field angular frequency. For the description of this system too we use the probability amplitude approach and expand the wave function of the system as

$$|\psi(t)\rangle = b_3(t)e^{i\delta t}|3,\{0\}\rangle + b_2(t)|2,\{0\}\rangle + \sum_{\lambda} b_{\lambda}(t)|0,\{\lambda\}\rangle + \sum_{\kappa} b_{\kappa}(t)|1,\{\kappa\}\rangle. \tag{23}$$

We substitute Eqs. (22) and (23) into the time-dependent Schrödinger equation and apply the elimination of the vacuum modes outlined in the previous section to obtain

$$i\dot{b}_3(t) = \delta b_3(t) + \Omega b_2(t), \tag{24}$$

$$i\dot{b}_2(t) = \Omega b_3(t) - i\frac{\gamma}{2}b_2(t) - i\int_0^t dt' b_2(t')K(t-t'), \tag{25}$$

$$i\dot{b}_{\lambda}(t) = g_{\lambda} b_2(t)e^{i(\omega_{\lambda}-\omega_{20})t}, \tag{26}$$

$$i\dot{b}_{\kappa}(t) = g_{\kappa} b_2(t)e^{i(\omega_{\kappa}-\omega_{21})t}. \tag{27}$$

The long time spontaneous emission spectrum in the Markovian reservoir is given by $S(\delta_{\lambda}) \propto |b_{\lambda}(t\to\infty)|^2$, and is calculated in a closed form with the use of the Laplace transform and the final value theorem [26] as

$$S(\delta_{\lambda}) \propto \gamma\left|\frac{(\delta_{\lambda}-\delta)b_2(0) + \Omega b_3(0)}{(\delta_{\lambda}-\delta)\left[\delta_{\lambda} + i\gamma/2 + i\tilde{K}(s\to -i\delta_{\lambda})\right] - \Omega^2}\right|^2. \tag{28}$$

The spectrum depends, in this case too, from the Laplace transform of the kernel, which in turn depends on the density of modes of the modified reservoir [see Eq. (11)]. We use the four models of the density of modes described in the previous section to calculate the above spectrum.

In the case of an isotropic photonic band gap material, with the use of Eq. (13) we find,

$$S(\delta_{\lambda}) \propto \gamma\left|\frac{(\delta_{\lambda}-\delta)b_2(0) + \Omega b_3(0)}{(\delta_{\lambda}-\delta)(\delta_{\lambda} + i\gamma/2) + i g^{3/2}\dfrac{\delta_{\lambda}-\delta}{\sqrt{\delta_{\lambda}-\delta_g}} - \Omega^2}\right|^2. \tag{29}$$

Let us assume first that the atom starts from its ground state, i.e. $|b_3(0)|^2 = 1$, $|b_2(0)|^2 = 0$. Then, the spectrum obtains a dark line at $\delta_{\lambda} = \delta_g$, except in the case that $\delta = \delta_g$, so in the case that the laser detuning becomes equal to the detuning from the band edge no dark line occurs, if the system is initially in state $|3\rangle$. We note that in the case when the transition $|2\rangle \to |1\rangle$ occurs in a Markovian reservoir, then no dark line appears in the spectrum [21,24]. If now the atom starts in a superposition of the ground ($|3\rangle$) and excited states ($|2\rangle$) with real expansion coefficients $[b_i(0)]$ then two dark lines can appear in the spectrum. The first dark line appears at

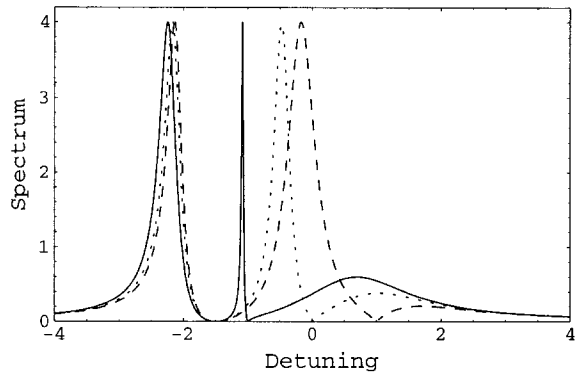

Fig. 6. The spontaneous emission spectrum $S(\delta_\lambda)$ given by Eq. (29) for parameters $g = 1$, $\Omega = 1$, $\delta = -1.5$, $b_2(0) = 1$, and $\delta_g = 0$ (dotted curve); $\delta_g = 1$ (dashed curve); $\delta_g = -1$ (full curve). All parameters are in units of γ.

$\delta_\lambda = \delta - \Omega b_3(0)/b_2(0)$ and is attributed to the laser-atom interaction [24]. The second dark line occurs at $\delta_\lambda = \delta_g$ and is attributed to the interaction with the non-Markovian reservoir. In this case too, the second dark line disappears if $\delta = \delta_g$. The behaviour of the above spectrum for the case that the system is initially in state $|2\rangle$ is shown in Fig. 6. We have chosen $\delta_g \neq \delta$ therefore, as predicted, two dark lines appears in the spectrum. We note the similarity of this spectrum and that of Fig. 4. This should be expected as the delta function density of modes (used in Fig. 4) represents a pure Jaynes–Cummings interaction [27].

If the more general, smoothed density of states of Eq. (15) is used (with $\epsilon \neq 0$), the spontaneous emission spectrum is given by

$$S(\delta_\lambda) \propto \gamma \left| \frac{(\delta_\lambda - \delta) b_2(0) + \Omega b_3(0)}{(\delta_\lambda - \delta)(\delta_\lambda + i\gamma/2) + i g^{3/2} \dfrac{\delta_\lambda - \delta}{i\sqrt{\epsilon} + \sqrt{\delta_\lambda - \delta_g}} - \Omega^2} \right|^2 . \tag{30}$$

Then, in the case that the atom starts in the ground state no dark line exists in the spectrum. However, if the atom starts in a superposition of the ground and excited states (with real expansion coefficients) the laser-induced dark line appears at the same frequency as before, i.e. at $\delta_\lambda = \delta - \Omega b_3(0)/b_2(0)$. This is verified in Fig. 7, where only a single dark line appears in the spectrum at $\delta_\lambda = \delta$, as $b_3(0) = 0$.

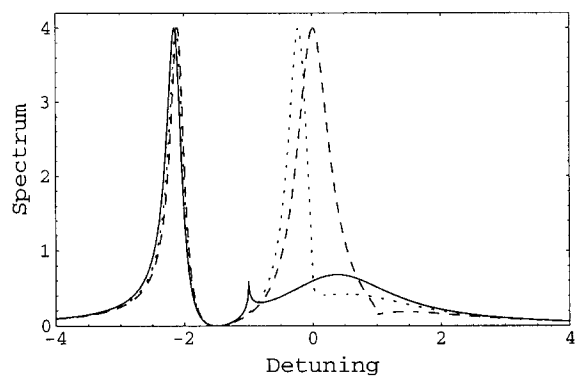

Fig. 7. The spontaneous emission spectrum $S(\delta_\lambda)$ given by Eq. (30) for parameters $g = 1$, $\epsilon = 0.3$, $\Omega = 1$, $\delta = -1.5$, $b_2(0) = 1$, and $\delta_g = 0$ (dotted curve); $\delta_g = 1$ (dashed curve); $\delta_g = -1$ (full curve).

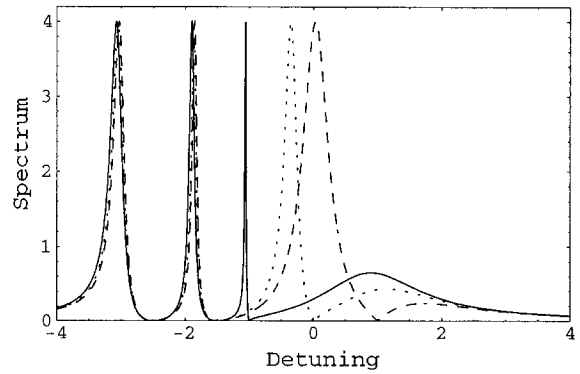

Fig. 8. The spontaneous emission spectrum $S(\delta_\lambda)$ given by Eq. (31) for parameters $g=1$, $g_1=1$, $\delta_c=-2.5$, $\Omega=1$, $\delta=-1.5$, $b_2(0)=1$, and $\delta_g=0$ (dotted curve); $\delta_g=1$ (dashed curve); $\delta_g=-1$ (full curve).

In the case of an isotropic photonic band gap with a defect with delta function density of modes then the spectrum reads

$$S(\delta_\lambda) \propto \gamma \left| \frac{(\delta_\lambda-\delta)b_2(0)+\Omega b_3(0)}{(\delta_\lambda-\delta)(\delta_\lambda+\mathrm{i}\gamma/2)+\mathrm{i}g^{3/2}\dfrac{\delta_\lambda-\delta}{\sqrt{\delta_\lambda-\delta_g}}-g_1^2\dfrac{\delta_\lambda-\delta}{\delta_\lambda-\delta_c}-\Omega^2} \right|^2 . \tag{31}$$

and one obtains, in general, two dark lines if the atom starts from the ground state, one at $\delta_\lambda=\delta_g$ and the other at $\delta_\lambda=\delta_c$. Any of these dark lines can disappear if either $\delta=\delta_g$, or $\delta=\delta_c$ so the spectrum exhibits only one dark line in this case. If now the atom starts in a superposition of the ground ($|3\rangle$) and excited states ($|2\rangle$) with real expansion coefficients then a third dark line can appear in the spectrum at $\delta_\lambda=\delta-\Omega b_3(0)/b_2(0)$. The spectrum for the case that the atom is initially in the excited state $|2\rangle$ (as in the previous cases), is displayed in Fig. 8. A very rich behaviour of the spectrum is found. The spectrum now has three dark lines at the predicted values and four different peaks can be seen.

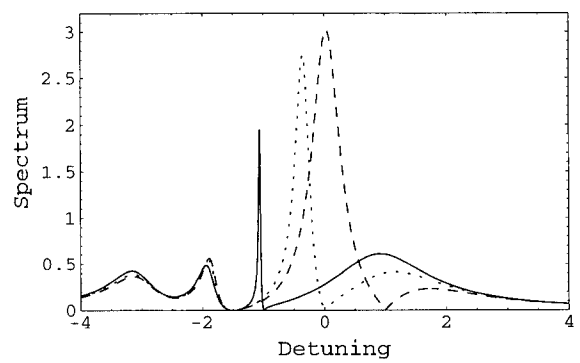

Fig. 9. The spontaneous emission spectrum $S(\delta_\lambda)$ given by Eq. (32) for parameters $g=1$, $g_1=1$, $\gamma_c=1$, $\delta_c=-2.5$, $\Omega=1$, $\delta=-1.5$, $b_2(0)=1$, and $\delta_g=0$ (dotted curve); $\delta_g=1$ (dashed curve); $\delta_g=-1$ (full curve).

Finally, in the case of an isotropic photonic band gap with a defect with Lorentzian density of modes the spectrum gets the form,

$$S(\delta_\lambda) \propto \gamma \left| \frac{(\delta_\lambda - \delta) b_2(0) + \Omega b_3(0)}{(\delta_\lambda - \delta)(\delta_\lambda + i\gamma/2) + i g^{3/2} \frac{\delta_\lambda - \delta}{\sqrt{\delta_\lambda - \delta_g}} + i g_1^2 \frac{\delta_\lambda - \delta}{i(\delta_c - \delta_\lambda) + \gamma_c/2} - \Omega^2} \right|^2 . \tag{32}$$

The dark lines in the spectrum appear at the same frequencies as those of the simple isotropic photonic band gap model, discussed above. However, the shape of the spectrum in this case differs from that of Fig. 6 as it is shown in Fig. 9.

4. Summary

In this article we have investigated the spontaneous emission properties of two distinct atomic models with one transition coupling to a Markovian reservoir while another transition coupling to a non-Markovian reservoir. Of specific interest to us were the existence of dark lines in the Markovian spontaneous emission spectrum. We have shown that dark lines can occur if the non-Markovian reservoir is described by certain densities of modes. In the case of the laser-driven scheme of Fig. 1(b) laser-induced dark lines can co-exist with non-Markovian reservoir-induced dark lines. Overall, spontaneous emission in the Markovian transition can be efficiently controlled (and even suppressed) by appropriately engineering the density of modes of the non-Markovian reservoir.

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